Improvements for the Partition Function of a Many Body System

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Based on the path integral approach the partition function $\mathcal{Z}$ of a many body system with separable two body interaction can be calculated in the sense of a semiclassical approximation. The widely used perturbed static path approximation (PSPA) [1,2] breaks down near a crossover temperature T_0 due to instabilities of the classical mean field solution. Improvements beyond the PSPA for $T \approx T_0$ and $T \ll T_0$ are desireable in order to get reasonable approximations for many body systems in the low temperature range where usual Monte Carlo techniques become very expensive. This is particularly true as often the critical temperature T_c of phase transitions is of the same order of magnitude as the crossover temperature T_0 where the PSPA breaks down.

The PSPA can be improved systematically within the crossover region by taking into account large non-Gaussian fluctuations [3]. Doing so an approximation applicable down to very low temperatures has been proposed [4]. These findings have been tested against exact results for the archetypical cases of a particle moving in a one dimensional anharmonic potential with and without barrier and the exactly solvable Lipkin-Meshkov-Glick model [5] for many body systems. The extensions may have potential applications in finite systems at low temperatures as in nuclear physics [6,7] and mesoscopic systems, e.g. for gap fluctuations in nanoscale superconducting devices previously studied within a PSPA type of approximation [8,9].

Fig. 1 shows the behaviour of the free energy $\mathcal{F}(\beta) = -T \ln \mathcal{Z}(\beta)$ as a function of the inverse temperature $\beta = 1/T$.

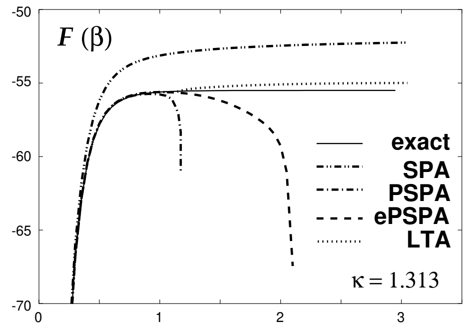

Fig. 1: $\mathcal{F}(\beta)$ of the Lipkin-Meshkov-Glick model computed for various approximations. For the chosen parameters a phase transition occurs at $\beta_c = 0.4\,\mathrm{MeV}^{-1}$ and the inverse crossover temperature is $\beta_0 = 1.1392\,\mathrm{MeV}^{-1}$.

As expected, pure static path approximation (SPA) deviates strongly from the exact results with decreasing temperature. The inclusion of quantum effects on the RPA level in the conventional PSPA improves the results a lot as long as β is not too close to the breakdown value β_0. Here, the conventional PSPA fails as the correction factor diverges. Instead, the "extended PSPA" (ePSPA) proposed in [4] smoothly passes the crossover region and agrees well with exact results even for inverse temperatures slightly above β_0 but definitely breaks down near $\beta = 2\beta_0$. A low temperature approximation (LTA), also proposed in [4], smoothly matches the ePSPA somewhat below β_0 and is able to even cover the range $\beta \gg \beta_0$ with astonishing accuracy. There only mean fields with small quantum fluctuations around the minima of the *intrinsic* static free energy are taken to contribute.

The development of the PSPA was motivated by the need for calculating nuclear level densities as a function of the excitation energy $\mathcal{E}^*$. Given the partition function this can be achieved by an inverse Laplace transform. Within a saddle point approximation (Darwin-Fowler method) one finds the behaviour for the level density shown in fig. 2:

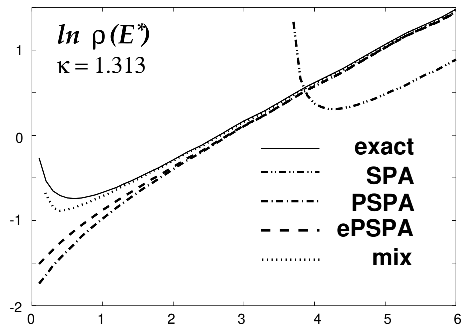

Fig. 2: $\ln \rho(\mathcal{E}^*)$ of the Lipkin-Meshkov-Glick model.

The conventional and extended PSPA results agree with the exact ones much better than those of pure SPA, which completely fails for $\mathcal{E}^* \lesssim 4\,\mathrm{MeV}$. Also shown in fig. 2 is a mixing of all types of approximations discussed in [4]. This procedure improves the conventional and extended PSPA considerably at low excitations. Very remarkably, it even describes the bending up of the exact level density qualitatively correct.

References

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