Self-Consistent Quantal Treatment of Decay Rates for Damped Systems

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Commonly quantum versions of the decay rate of damped meta-stable systems are treated on the basis of the Caldeira-Leggett model [1,2]. Unfortunately, this model assumes some simplified coupling to a linear heat bath. Furthermore it does not make any predictions about the input of such important quantities as the potential energy and the inertia, which need to be chosen on an entire phenomenological level. These features do not allow the model to be applied to self-bound Fermi systems. There, one would like to see the collective variables introduced in some self-consistent fashion, with a microscopic treatment of *all* transport coefficients alike.

One possible attempt to overcome these deficiencies is formulated in [3] in connection to nuclear physics. It is based on a quantal transport equation which is derived within a locally harmonic approximation exploiting linear response theory. A transport equation necessarily describes evolution in real time. Therefore, in barrier regions quantum effects can be accounted for only above a critical temperature T_c, which is larger than the so called "crossover temperature" T_0 which one encounters for imaginary time propagation [4]. Another disadvantage of the derivation of the transport equation mentioned is that it bases on the deformed shell model. As one does not start from a genuine two body interaction, self-consistency is handled on a semi-microscopic level only.

First steps to overcome these deficiencies have been undertaken in [5] by adapting a previously developed formalism to evaluate the partition function for bound systems with separable two body interactions. One starts from the static path approximation (SPA) functional integrals as an approximation to the classical or high temperature limit. Then small scale fluctuations around this static path are treated to second order, such that quantum effects come in through local RPA. In the literature this is often referred to as the perturbed static path approximation (PSPA) [6,7].

The simplest Hamiltonian which allows one to introduce collective degrees of freedom self-consistently is of the following structure

$$\hat{H} = \hat{H}_0 + \frac{k}{2}\hat{F}\hat{F} \,, \tag{1}$$

with one body operators $\hat{H}_0$ and $\hat{F}$. The product $\hat{F}\hat{F}$ mimics an effective separable two body interaction. For isoscalar modes the coupling constant k is *negative*. The ansatz (1) is considered to define a microscopic model for just that collective mode we want to address to. The latter, here is related to the average of $\hat{F}$.

An extension of the PSPA has been found that allows to describe also strongly damped phenomena [5]. To this end we have successfully modified the basic ideas of the Caldeira-Leggett model such that they can be taken over to nuclear problems. Like in this model, dissipation is taken into account in a simple way: The discrete spectral density of the nucleonic heat bath is replaced by a continous one. This is justified at not too low temperatures and is meant to simulate the presence of interactions residual to the mean field. These interactions couple mean field states to much more complicated ones. Finally the nucleonic response function $\chi(q,\omega)$ is replaced by the one of the local damped harmonic oscillator

$$\chi\mathrm{osc}(q,\omega) = \frac{-1}{M(q)}\,\frac{1}{\omega^2 + i\Gamma(q)\omega - \Omega^2(q)} \tag{2}$$

and the transport coefficients for inertia $M(q)$, friction $\gamma(q) = M(q)\Gamma(q)$ and stiffness $C(q) = M(q)\Omega^2(q)$ can be extracted from the microscopic theory instead of being restricted to macroscopic input.

An expression for the decay rate of a metastable system like an excited heavy nucleus can be derived from the imaginary part of the free energy [8] after evaluating a path integral in the saddle point approximation. Above the crossover temperature T_0 where the PSPA formalism breaks down our result for the decay rate factorizes into the classical expression derived by Kramers in 1940 [9] and two correction factors [5]: One of them accounts for the inertias being different at the barrier q_b and the minimum q_a:

$$\sqrt{\frac{M_a}{M_b}} \tag{3}$$

This factor is a generalization of $M_a = M_b \equiv M$ used in [1,2]. The second correction factor represents the quantum corrections to "thermal hopping"

$$f_{\mathrm{qm}}^{\mathrm{PSPA}} = \prod_{r>0} \frac{\nu_r^2 + \Gamma_b\,\nu_r + \Omega_b^2}{\nu_r^2 + \Gamma_b\,\nu_r + \varpi_b^2} \,:\, \prod_{r>0} \frac{\nu_r^2 + \Gamma_a\,\nu_r + \Omega_a}{\nu_r^2 + \Gamma_a\,\nu_r + \varpi_a^2} \tag{4}$$

where $\nu_r = 2\pi T r$ are the Matsubara frequencies and $\varpi(q)$ are the local collective frequencies (at the barrier: $\varpi^2(q_b) < 0$). The nice feature about this structure is that it converges for *all* conceivable values of the transport coefficients as long as $T > T_0$ where $T_0 < T_c$.

References

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