Combination of LEP 1 S–Matrix Results

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The process $e^+e^- \to f\bar{f}$ at LEP 1 can be mediated in the s-channel by two spin-1 bosons, a massless photon and a massive Z boson. The cross-sections and asymmetries of these reactions can be split up into contributions from γ exchange, the Z resonance terms and γ/Z interference terms.

For the conventional analysis of the LEP 1 data only the Z resonance terms are determined as independent parameters in a fit to the measured cross-sections and asymmetries. The contributions from γ exchange and γ/Z interference are in general constrained to the Standard Model prediction.

An alternative phenomenological approach is the S–matrix formalism, where contributions from boson exchange and interference are explicitly allowed to vary independently. The resulting parametrisations of the fermion-pair cross-section $\sigma^0_{\rm tot}(s)$ and asymmetry $A^0_{\rm fb}(s)$ in lowest order are given by

$$\sigma^0_a(s) = \frac{4}{3}\pi\alpha^2 \left[\frac{g^a_f}{s} + \frac{j^a_f(s-\bar{m}^2_{\rm Z}) + r^a_f s}{(s-\bar{m}^2_{\rm Z})^2 + \bar{m}^2_{\rm Z}\bar{\Gamma}^2_{\rm Z}} \right] \; ;$$

$$a = {\rm tot, fb} \quad \text{and} \quad A^0_{\rm fb}(s) = \frac{3}{4}\,\frac{\sigma^0_{\rm fb}(s)}{\sigma^0_{\rm tot}(s)}$$

Here α is the fine structure constant and $\sqrt{s}$ the centre-of-mass energy. Besides the Z mass $\bar{m}_{\rm Z}$ and width $\bar{\Gamma}_{\rm Z}$ there are six parameters per final state fermion f, three for the cross-sections (a =tot) and three for the asymmetries (a =fb). The photon exchange is described by $g^{\rm tot}_f$ and $g^{\rm fb}_f (\equiv 0)$ and is assumed to be known. This leaves four parameters, namely r^a_f describing the Z exchange and j^a_f for the interference. The hadronic asymmetry and hence $r^{\rm fb}_{\rm had}$ and $j^{\rm fb}_{\rm had}$ are not measured.

The free parametrisation of the interference, especially for the hadronic cross-section, leads to a large uncertainty on the Z mass when using LEP1 data alone. This is because the interference increases the cross-sections at energies above the peak and decreases them at lower energies. Therefore a change in this contribution effectively shifts the position of the peak, which can be interpreted as a change in the Z mass. Since the hadronic channel provides data with the highest statistics and hence the smallest cross-section error it dominates the determination of the Z mass. This leads to a strong anti-correlation between m_Z and $j^{\rm tot}_{\rm had}$ as illustrated in fig. 1.

Including data taken further away from the Z peak yields a much better measurement of the interference. LEP 2 data taken at energies between 130–209 GeV significantly reduce the correlation between m_Z and $j^{\rm tot}_{\rm had}$ and the corresponding errors, as shown in fig. 1.

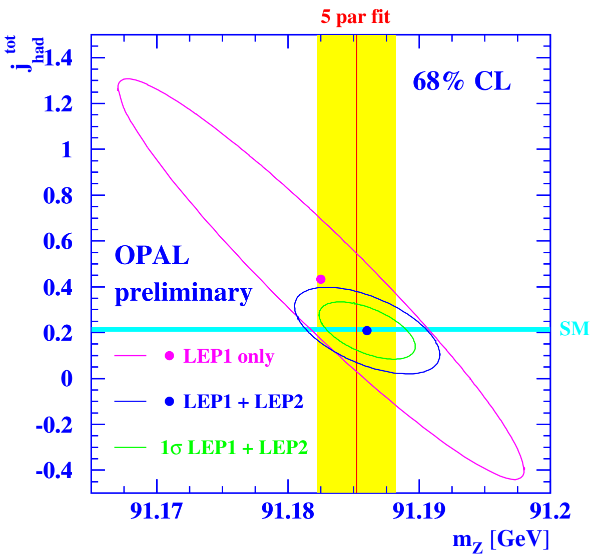

Fig. 1: Contours in the m_Z-$j^{\rm tot}_{\rm had}$-plane for fits to LEP 1 and LEP 2 data of the OPAL experiment [1]. The horizontal band indicates the Standard Model prediction for $j^{\rm tot}_{\rm had}$. The vertical band is the m_Z error for a fit fixing $j^{\rm tot}_{\rm had}$ to this value.

All the four LEP experiments, ALEPH, DELPHI, L3 and OPAL, performed such analyses in terms of S–matrix parameters, but the combination of the results of the four experiments is still in progress.

This combination is non-trivial, since on the one hand there are uncertainties, such as the beam energy or the t-channel corrections for $e^+e^- \to e^+e^-$ scattering, which have complex correlations between the experiments. And on the other hand these uncertainties are in general only known for the input data of the fit, the measured cross-sections and asymmetries, but not for the output, the fit parameters. To determine the common errors for the S–matrix fit parameters the same techniques are used as for the LEP combination of the Z^0 lineshape results [2]. The common errors determined so far are in most cases consistent with the common errors determined in [2], but there are also significant differences. For example, the m_Z error due to the LEP beam energy determination results in 3.1 MeV for the S–matrix fit, much larger than the 1.7 MeV determined for the lineshape parameters with fixed γ/Z interference. The difference is due to the fact that for the S–matrix m_Z measurement the cross-sections measured at energies further away from the peak (± 3 GeV) contribute more strongly. This data was taken in the early years of LEP 1 when the LEP beam energy error was substantially larger.

References

[1] OPAL Collaboration, Determination of S-matrix parameters from fermion-pair production at OPAL, OPAL PN474, July 2001.

[2] Combination procedure for the precise determination of Z boson parameters from results of the LEP experiments, CERN-EP-2000-153, December 2000.