Chiral Dynamics and Pionic States of Pb and Sn Isotopes ◇

E.E. Kolomeitsev [a,b], N. Kaiser, and W. Weise [a,c]
[a] ECT*, Trento [b] NBI, Copenhagen [c] TU München, Garching

Recent accurate data on $1s$ states of π^- bound to Pb [1] and Sn [2] isotopes have set new standards and constraints for the detailed analysis of s-wave pion-nucleon interactions. This topic has a long history culminating in various attempts to understand the notorious "missing repulsion" in the π-nucleus interaction: the standard ansatz for the (energy independent) s-wave pion-nuclear optical potential, given in terms of the empirical threshold πN amplitudes times densities $\rho_{p,n}$ and supplemented by sizable double-scattering corrections, still misses the observed repulsive interaction by a large amount. This problem has traditionally been circumvented on purely phenomenological grounds by introducing an extraordinarily large repulsive real part ($\mathrm{Re}\, B_0$) in the ρ^2 terms of the π-nucleus potential. The arbitrariness of this procedure is of course unsatisfactory.

In recent papers [3,4] we have re-investigated this issue from the point of view of the distinct explicit energy dependence of the pion-nuclear polarization operator [3]. The starting point is the energy- and momentum-dependent polarization operator $\Pi(\omega, \vec{q}; \rho_p, \rho_n)$. In the limit of very low proton and neutron densities, $\rho_{p,n}$, the pion self-energy reduces to $\Pi = -(T^+ \rho + T^- \delta\rho)$ with $\rho = \rho_p + \rho_n$ and $\delta\rho = \rho_p - \rho_n$, where $T^\pm$ are the isospin-even and isospin-odd off-shell πN amplitudes. Terms of non-leading order in density (double scattering (Pauli) corrections of order $\rho^{4/3}$, absorption effects of order ρ^2 etc.) are known to be important. We incorporate them within in-medium chiral effective field theory, following [4]. Double scattering corrections are fully incorporated at 2-loop order. Absorption effects and corresponding dispersive corrections appear at the 3-loop level and through short-distance dynamics parameterized by contact terms, not explicitly calculable within the effective low-energy theory. The imaginary parts associated with these terms are well constrained by the systematics of observed widths of pionic atom levels throughout the periodic table. The real part of the s-wave absorption term ($\mathrm{Re}\, B_0$) is still the primary source of theoretical uncertainty. In practice, our strategy is to start from $\mathrm{Re}\, B_0 = 0$ (as suggested also by the detailed analysis of the pion-deuteron scattering length) and then discuss the possible error band induced by varying $\mathrm{Re}\, B_0$ within reasonable limits [3]. The canonical parameterization of p-wave parts is included as well.

We proceed by using the local density approximation and solve the Klein-Gordon equation (KGE) $[(\omega - V_c)^2 + \vec{\nabla}^2 - m_\pi^2 - \Pi(\omega - V_c; \rho)]\phi = 0$. Note that the explicit energy dependence of Π requires that the Coulomb potential V_c must be introduced in the canonical gauge invariant way wherever the pion energy ω appears. With input specified in details in [3], we have solved KGE with the explicitly energy dependent pion self-energy. The results for the binding energies and widths of $1s$ and $2p$ states in pionic ^{205}Pb are shown in Figure (triangles in the upper plot). Also shown for comparison is the outcome of a calculations using a "standard" phenomenological (energy independent) s-wave optical potential (empty circles). This approach fails and shows the "missing repulsion" syndrome, leading to a substantial overestimate of the widths. The explicit energy dependence in $T^\pm$ provides a required remedy against it: the replacement $\omega \to \omega - V_c > m_\pi$ increases the repulsion in T^- and disbalances the "accidental" cancellation between the πN sigma term σ_N and the range term proportional to ω^2 in T^+, such that $T^+(\omega - V_c) < 0$ (repulsive).

Uncertainties in $\mathrm{Re}\, B_0$, in the radius and shape of the neutron density distribution, and in the input for the sigma term σ_N have been analysed in [3]. Their combined effect falls within the experimental errors in Figure (upper plot).

Using the same (explicitly energy dependent) scheme we have predicted binding energies and widths for pionic $1s$ states bound to a chain of Sn isotopes. Results are shown in Figure (lower plot) in comparison with the experimental data [2] reported after the calculations. This figure also gives an impression of the sensitivity with respect to variations of the (input) πN sigma term.

The leading-order equivalence of the present approach with the "chiral restoration" scenario is demonstrated and discussed in [3].

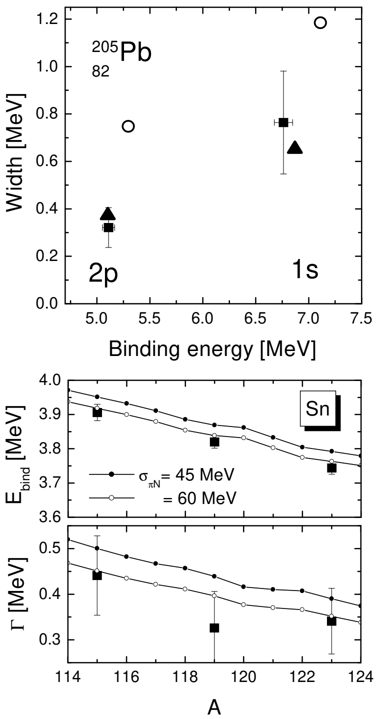

Figure: Upper plot: Binding energies and widths of pionic $1s$ and $2p$ states in ^{205}Pb isotopes. Full triangles correspond to the energy dependent s-wave polarization operator, whereas open circles correspond to the energy-independent one. Data are from [1]. Lower plot: Binding energies and widths of pionic $1s$ states in Sn isotopes. The curves show our results in comparison with the data [2]. The sensitivity to the πN sigma term (input) is also shown.

References

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