The Nucleon in a Box ◊

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State of the art lattice QCD simulations of the hadron spectrum utilizing fully dynamical fermions can only be performed at rather large quark masses. When setting the scale with the help of the Sommer parameter $r_0 = 0.5$ fm, the mass of the lowest lying 0^- meson is typically larger than 500 MeV in such simulations. Comparing this value with the mass of physical pions $m_\pi = 140$ MeV, it is obvious that chiral extrapolations—i.e. the dependence of an observable like the hadron mass on the quark-masses employed in the simulation—are an important issue in lattice QCD. The methods of Chiral Effective Field Theory can be utilized to address this question. In ref. [1] it was shown that the quark-mass dependence of the mass of the nucleon calculated to next-leading-leading one-loop order (*i.e.* $\mathcal{O}(p^4)$ in BChPT counting, cf. Fig. 1) provides an excellent description for the chiral extrapolation. Given that the quark-mass dependence of the pion mass (squared) scales linearly with the quark-mass over a wide range of mass-parameters as observed in lattice QCD simulations, the problem of the quark mass dependence of the mass of the nucleon can be simplified to the question of the pion mass dependence of the mass of the nucleon. In "free space" one obtains

$$\begin{aligned} M_N(\infty) &= M_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3 \\ &+ \left[e_1^r(\lambda) - \frac{3}{64\pi^2 f_\pi^2}\left(\frac{g_A^2}{m_0} - \frac{c_2}{2}\right) - \frac{3}{32\pi^2 f_\pi^2} \right. \\ &\times \left. \left(\frac{g_A^2}{m_0} - 8c_1 + c_2 + 4c_3\right) \ln\frac{m_\pi}{\lambda}\right] m_\pi^4 \\ &+ \frac{3g_A^2}{256\pi f_\pi^2 m_0^2} m_\pi^5 + O(m_\pi^6)\,. \end{aligned} \quad (1)$$

While most of the parameters in this expression are known from low energy physics [1], M_0 and e_1 are fit to a selected set of large volume data. The resulting extrapolation curve is shown in the upper part of Fig. 2. The analysis of ref. [2] suggests that Eq.(1) should provide a consistent representation of the chiral extrapolation function up to a pion mass of 600 MeV, in agreement with the result shown in Fig. 2.

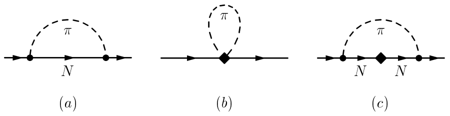

Fig. 1: Diagrams contributing to the quark mass dependence of the nucleon mass to $\mathcal{O}(p^3)$ (a) and to $\mathcal{O}(p^4)$ (b+c) in BChPT.

Evaluating the $\mathcal{O}(p^4)$ contributions to the nucleon mass (Fig. 1) now in a finite box of length L, one obtains the modified mass formula

$$M_N(L) = M_N(\infty) + \Delta^{(3)}(L) + \Delta^{(4)}(L) + O(p^5)\,. \quad (2)$$

We note that the mass-shifts $\Delta^{(n)}(L)$ both to $\mathcal{O}(p^3)$ (Eq.(3)) and to $\mathcal{O}(p^4)$ (Eq.(4)) only depend on parameters are already known from the infinite volume result (Eq.(1)):

$$\begin{aligned} \Delta^{(3)}(L) &= \frac{3g_A^2 M_0 m_\pi^2}{16\pi^2 f_\pi^2} \int_0^\infty \mathrm{d}x \sum_{\vec{n}}{}' \\ &\quad \times \mathbf{K_0}\left(L|\vec{n}|\sqrt{M_0^2 x^2 + m_\pi^2(1-x)}\right)\,, \quad (3) \\ \Delta^{(4)}(L) &= \frac{3m_\pi^4}{4\pi^2 f_\pi^2} \sum_{\vec{n}}{}' \left[(2c_1 - c_3)\frac{\mathbf{K_1}(L|\vec{n}|m_\pi)}{L|\vec{n}|m_\pi} \right. \\ &\quad \left. + c_2 \frac{\mathbf{K_2}(L|\vec{n}|m_\pi)}{(L|\vec{n}|m_\pi)^2}\right]\,. \quad (4) \end{aligned}$$

The result is expressed in terms $\mathbf{K_i}$-Bessel-functions [3], where the variable $|\vec{n}|$ counts how often a pion crosses the box-boundary (with periodic boundary conditions). The resulting L-, respectively volume-dependence of the nucleon mass is shown in the lower part of Fig. 2 for a given pion mass of 545 MeV. The $\mathcal{O}(p^4)$ curve provides an excellent prediction for the volume-dependence of the nucleon mass when compared to actual lattice QCD data, further examples for different pion masses are given in [3].

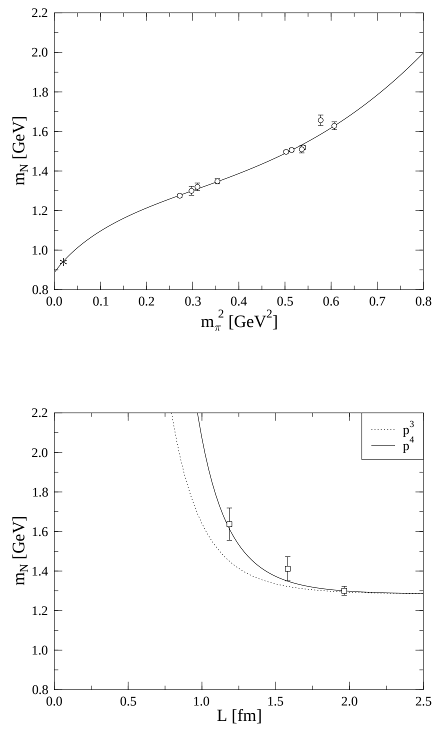

Figure 2: Upper plot: Fit of the $\mathcal{O}(p^4)$ BChPT extrapolation function of Eq.(1) to the set of large volume data. Lower plot: Resulting prediction for the volume dependence for a fixed pion mass of 545 MeV to $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$. Note the substantial increase of the nucleon mass for small box sizes.

References

[1] M. Procura, T.R. Hemmert and W. Weise, Phys. Rev. **D69** (2004) 034505
[2] V. Bernard, T.R. Hemmert and U.-G. Meißner, Nucl. Phys. **A732** (2004) 149
[3] A. Ali-Khan *et al.* (QCDSF-UKQCD collaboration), preprint no. [hep-lat/0312030]; submitted to Phys. Rev. **D**.

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