Chiral Extrapolation of Nucleon Properties from Lattice QCD ◇

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Lattice QCD on one side and chiral effective field theory on the other are progressively developing as important tools to deal with the nonperturbative nature of low-energy QCD and the structure of hadrons. At present, however, there is a gap between the relatively large quark masses accessible in fully dynamical lattice simulations of nucleon properties and the small quark masses relevant for comparison with physical observables. In our work we study a way to merge Chiral Perturbation Theory (χPT) which predicts the quark mass dependence of a nucleon observable and lattice simulations where the quark mass is a tunable parameter. We explore the feasibility of a systematic approach based on chiral effective Lagrangian for the extrapolation of nucleon properties from lattice QCD.

In [1] we have analyzed the quark mass expansion of the axial-vector coupling constant g_A of the nucleon. We compare two versions of nonrelativistic chiral effective field theory: One scheme restricted to pion and nucleon degrees of freedom only, and an alternative approach which incorporates explicit $\Delta(1230)$ resonance degrees of freedom. It turns out that, in order to approach the physical value of g_A in a leading one-loop calculation, the inclusion of the explicit $\Delta(1230)$ degrees of freedom is crucial. With information on important higher order couplings constrained from analyses of the $\pi N \to \pi\pi N$ reaction, a chiral extrapolation function $g_A(m_\pi)$ is obtained, which works well from the chiral limit across the physical point into the region of present lattice data (see Fig.1). The resulting enhancement of $g_A(m_\pi)$ near the physical pion mass is found to arise from an interplay between long- and short- distance physics.

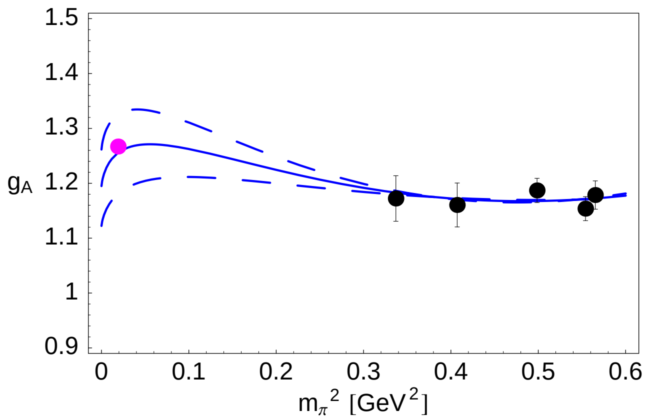

Figure 1: We incorporate known information from $\pi N \to \pi\pi N$ differential cross sections [2] into the leading one-loop analysis with explicit Δ degrees of freedom and fit the five lattice data points provided by QCDSF Collaboration [3]. The solid line corresponds to the central value of the relevant higher order couplings, whereas the dashed curves show the uncertainty in their determination.

The quark mass dependence of the nucleon mass M_N is the subject of [4]. An interpolation of this observable, between a selected set of fully dynamical two-flavor lattice QCD data and its physical value, is studied using relativistic baryon χPT up to order p^4 in the framework of infrared regularization [5]. In order to minimize uncertainties due to lattice discretization and finite volume effects our numerical analysis takes into account only simulations performed with lattice spacings $a < 0.15\,\mathrm{fm}$ and $m_\pi L > 5$. We have also restricted ourselves to data with $m_\pi < 600\,\mathrm{MeV}$ and $m_{\mathrm{sea}} = m_{\mathrm{val}}$. A good interpolation function is found already at one-loop level and chiral order p^3. We show that the next-to-leading one-loop corrections are small (see Fig.2). From the p^4 numerical analysis we deduce the nucleon mass in the chiral limit, $M_0 \approx 0.88\,\mathrm{GeV}$, and the pion-nucleon sigma term $\sigma_N = (49 \pm 3)\,\mathrm{MeV}$ at the physical value of the pion mass.

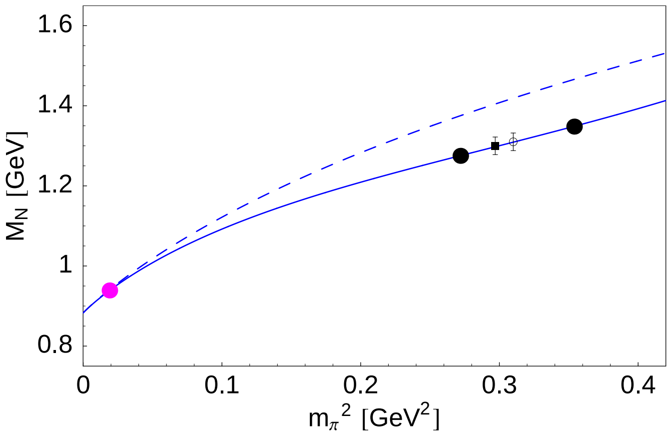

Figure 2: The solid curve is obtained by fitting the lattice data points (solid dots, boxes and empty symbols) provided by CP-PACS, JLQCD and QCDSF Collaborations [6] with the formula for the m_π dependence of M_N at chiral order p^4. We use as input the physical value of the nucleon mass. The dashed curve is the leading one-loop result using as parameters the central values from the previous fit.

References

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◇ Supported in part by BMBF and DFG