Nuclear Energy Density Functional from Chiral Pion-Nucleon Dynamics ◇

S. Fritsch, N. Kaiser, and W. Weise

Using in-medium chiral perturbation theory [1], we have calculated the energy per particle $\bar{E}(\rho)$ of symmetric nuclear matter and pure neutron matter up to fifth order in the small momenta (k_f and m_π). Contributions from interactions originate in our calculation exclusively from one-pion and two-pion exchange between nucleons. Several empirically known nuclear matter properties such its binding energy, saturation density, compressibility and asymmetry energy can be well reproduced with one single adjustable parameter, a momentum cut-off of $\Lambda \simeq 0.65\,\mathrm{GeV}$.

In a subsequent work [2] we have extended this approach to the inhomogenous many-nucleon system employing the density-matrix expansion of Negele and Vautherin [3]. This method allows one to compute diagrammatically the energy density functional in the form of a systematic gradient expansion. Going up to second order in density gradients $\vec{\nabla}\rho$ the energy density functional relevant for $N = Z$ even-even nuclei reads:

$$\mathcal{E}[\rho,\tau,\vec{J}] = \rho\bar{E}(\rho) + \left(\tau - \frac{3}{5}\rho k_f^2\right)\left[\frac{1}{2M} - \frac{5k_f^2}{56M^3} + F_\tau(\rho)\right] + (\vec{\nabla}\rho)^2 F_\nabla(\rho) + \vec{\nabla}\rho\cdot\vec{J}F_{\mathrm{SO}}(\rho) + \vec{J}^{\,2}F_J(\rho) \quad (1)$$

with the kinetic energy density $\tau(\vec{r})$ and the spin-orbit density $\vec{J}(\vec{r})$. The strength functions $F_{\tau,\nabla,\mathrm{SO},J}(\rho)$ arising from NN-interactions encode new dynamical information specific for inhomogeneous many-nucleon systems. In phenomenological Skyrme parameterizations $F_\tau(\rho)$ depends linearly on the density whereas $F_{\nabla,\mathrm{SO},J}(\rho)$ are just constants.

The expression in square brackets in eq. (1) has the interpretation of a reciprocal density dependent effective nucleon mass which is shown in fig. 1. The relatively strong density dependence of $\widetilde{M}^*(\rho)$ (and also of the other strength functions $F_{\tau,\nabla,\mathrm{SO},J}$) is caused in our calculation by the explicit presence of the small mass scale $m_\pi = 135\,\mathrm{MeV}$.

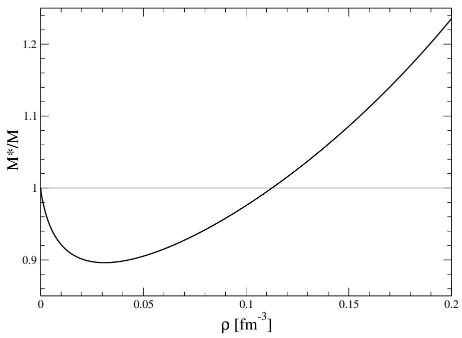

Fig. 1: The effective nucleon mass $\widetilde{M}^*(\rho)$ divided by the free nucleon mass M versus the nucleon density $\rho = 2k_f^3/3\pi^2$.

The nuclear surface energy is determined largely by the value of the strength function $F_\nabla(\rho)$ at about half nuclear matter saturation density $\rho_0/2 \simeq 0.08\,\mathrm{fm}^{-3}$. As can be seen in fig. 2 our present calculation gives a noticably higher value for $F_\nabla(\rho)$ at that point in comparison to Skyrme parameterizations. Two-pion exchange with explicit Δ-isobar excitation can somewhat reduce the strength of the $(\vec{\nabla}\rho)^2$-term. Work along this line is in progress.

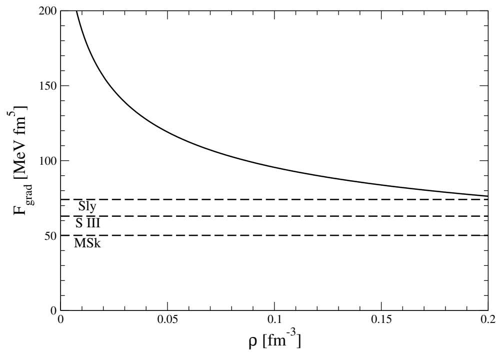

Fig. 2: The strength function $F_\nabla(\rho)$ versus nucleon density. The three horizontal dashed lines show the constant values of various Skyrme forces.

The full line in fig. 3 shows the result of iterated 1π-exchange for the strength function $F_{\mathrm{SO}}(\rho)$ belonging to the spin-orbit term in the nuclear energy density functional. Compared to the "empirical" value $F_{\mathrm{SO}}(\rho) \simeq 90\,\mathrm{MeVfm}^5$ the spin-orbit coupling generated by iterated 1π-exchange has the right magnitude but however the wrong sign. Note that this spin-orbit coupling is not a relativistic effect, but instead it is linearly proportional to the nucleon mass M. The dashed-dotted line in fig. 3 shows the relatively small contribution to $F_{\mathrm{SO}}(\rho)$ from irreducible 2π-exchange (proportional to M^{-1}). Actually, when combined with further sizeable 2π-exchange three-body spin-orbit terms [4] most of the "wrong-sign" spin-orbit coupling gets cancelled. Lorentz scalar and vector mean-fields with their in-medium behavior governed by QCD sum rule could therefore provide the appropriate dynamical framework for the large nuclear spin-orbit coupling.

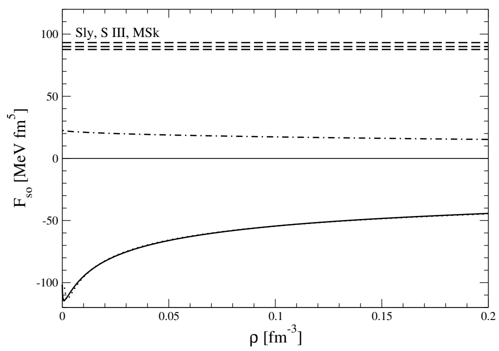

Fig. 3: The strength function $F_{\mathrm{SO}}(\rho)$ versus nucleon density. The three horizontal dashed lines show the constant values of various Skyrme forces.

References

[1] N. Kaiser, S. Fritsch, W. Weise, Nucl. Phys. **A697** (2002) 255
[2] N. Kaiser, S. Fritsch, W. Weise, Nucl. Phys. **A724** (2003) 47
[3] J.W. Negele, D. Vautherin, Phys. Ref **C5** (1972) 1472
[4] N. Kaiser, Phys. Rev. **C68** (2003) 054001

◇ Supported in part by BMBF, GSI and DFG.