Thermodynamics of Two-Colour QCD and the NJL Model ◊

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The phase structure of QCD has been subject of intense investigation in recent years. Precise numerical data have become available concerning QCD thermodynamics at high temperature via large-scale computer simulations on the lattice. The study of full QCD at finite baryon density is still a formidable challenge, due to the limitations of standard Monte Carlo techniques when applied to systems at finite density. A non-zero chemical potential leads to a complex determinant of the Euclidean Dirac operator and results in massive cancellations among configurations. There is increasing activity in this sector, aimed at finding alternative computation methods [1]. One response to this situation is to start by exploring simpler QCD-like theories with additional antiunitary symmetries that guarantee the Fermion determinant to be real and therefore allow the study of such theories on the lattice. Examples of such theories include QCD with two colours and fundamental quarks [2], and QCD with an arbitray number of colours and adjoint quarks.

In two-colour QCD, diquarks can form colour singlets which are the baryons of the theory. The lightest baryons and the lightest quark-antiquark excitations (pions) have a common mass, m_π, and this spectrum determines the properties of the ground state for small chemical potentials. General arguments predict a phase transition from the vacuum to a state with finite baryon density at a critical chemical potential $\mu_c = m_\pi/2$.

We have explored the relationship between $N_c = 2$ QCD and a corresponding Nambu and Jona-Lasinio model in which gluonic degrees of freedom are "integrated out" and replaced by a local four-point interaction of quark colour currents. Our specific aim was to test the effectiveness of the NJL model in reproducing the main features predicted for two-colour QCD, and to compare our results quantitatively with those obtained from recent lattice calculations. We started from the standard colour current interaction, and Fierz transformed it in order to write it explicitly in terms of the quark-quark and quark-antiquark interactions, whose couplings turned out to be equal. We have then bosonized our Lagrangian, in order to work directly in terms of the fields describing the chiral (σ) and diquark (Δ) condensates. By minimizing the thermodynamical potential of the system in the mean field approximation, we have studied the behaviour of the chiral and diquark condensates, and of the baryon density, as functions of temperature T and baryon chemical potential μ, both in the chiral limit and for finite values of the current quark masses. We have investigated, again for both zero and finite quark masses, the two-colour QCD phase diagram in the T-μ plane. As further applications we evaluated the pion, sigma and scalar diquark masses, and the pion decay constant, as functions of the temperature and of the chemical potential. We confirm that a second order phase transition takes place at $\mu_c = m_\pi/2$, above which diquarks can condense, and correspondingly the chiral condensate is reduced. We find a very good agreement with the data obtained by lattice simulations, as we can see from the upper Figure.

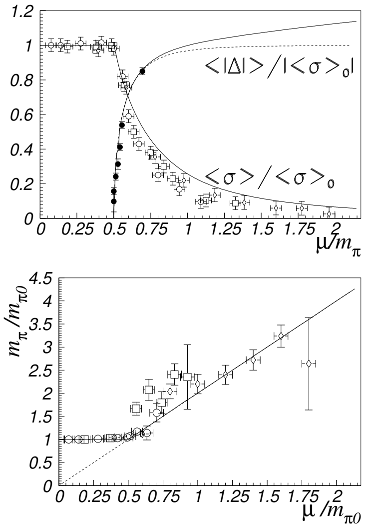

Figure: Upper plot: Scaled $\langle\sigma\rangle$ and $\langle|\Delta|\rangle$ as functions of the chemical potential at $T = 0$: our results (solid lines) are compared to the lattice data taken from ref. [3]. Lower plot: Scaled pion mass as a function of $\mu/m_{\pi 0}$ at $T = 0$ (continuous line), compared to lattice data [3]. In both plots, the dashed lines are the predictions coming from [4].

By considering the quark-antiquark correlation function in the Bethe-Salpeter equation, we are then able to evaluate the pion and σ masses, and the pion decay constant, as functions of temperature and chemical potential. Lattice data are available only for the pion mass, for which we show the results in the lower Figure: again our curve is in very good agreement with the lattice data.

In the chiral limit, and at $\mu = 0$, we find that the theory is invariant under an enlarged symmetry: in this limit, the thermodynamical potential depends only on $\sigma^2 + \Delta^2$. This is the so-called Pauli-Gürsey symmetry, a characteristic feature of two-colour QCD which comes out naturally in our model. As soon as we consider a finite chemical potential, even if very small, the favourable phase is the one with zero chiral condensate and nonzero diquark condensate. This means that in the chiral limit diquarks can always condense in two-colour QCD: the pion mass is equal to zero in this limit, so that the critical chemical potential, equal to half the pion mass, is also zero, and therefore the system is always in the superfluid phase.

Our work has therefore shown that it is possible to translate two-colour QCD into a corresponding NJL model which can reproduce all the main features of $N_c = 2$ QCD. The agreement that we find with the lattice data is remarkable. In the chiral limit the model exibits the Pauli-Gürsey symmetry and all the other characteristics predicted for this theory.

References

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◊ Supported in part by BMBF and INFN