All-Order Low-Energy Expansion in the 3-Body System ◇

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In "pion-less" Effective Field Theory, the power-counting in the 3-body system with a shallow two-body bound state was extended and systematised to all orders in the low-energy expansion in Ref. [1]. The analytical considerations rest on the tenet that a 3-body force (3BF) is included if and only if necessary to cancel cut-off dependences in observables at a given order. One momentum-independent 3BF is needed for cut-off independence already at leading order (LO), and still at next-to-leading order (NLO), in the $^2\mathrm{S}_{\frac{1}{2}}$ (triton) channel, see Fig. 1. It is determined by the nd scattering length in that channel. One and only one new parameter enters at next-to-next-to-leading order ($\mathrm{N^2LO}$) in this channel, namely the Wigner-$SU(4)$-symmetric 3BF with two derivatives,. It is fixed to reproduce the triton binding energy. Only the $SU(4)$-symmetric 3-body forces are systematically enhanced over their contributions found from a naïve dimensional estimate, with a $2n$-derivative 3BF entering at the $2n$th order. A new, computationally convenient and simple scheme to perform higher-order calculations in the three-body system iterates a kernel to all orders, which has been expanded to the desired order of accuracy. The nd scattering phase shifts agree well with phase shift analysis and modern potential model calculations, see Fig. 2.

Thus, calculations of triton and ^{3}He properties at very low energies are now possible with an accuracy of better than 1%, as can be estimated from the systematic power counting developed. This uncertainty on the theory side is considerably larger than the experimentally induced one. Albeit the triton binding energy is known to very high accuracy, the nd scattering length in this channel, $a_2 = (0.64 \pm 0.04)$ fm, is determined only to $\sim 6\%$ accuracy. This is particularly damaging for high-accuracy calculations at low energies involving the triton or ^{3}He wave function, as necessary e.g. for fundamental cross sections in neutrino-physics and big bang nucleo-synthesis. To cure this problem, an experiment at the Paul-Scherer-Institut will improve the experimental accuracy of this scattering length by one order of magnitude [2].

In contradistinction, Wigner-symmetric 3BFs are suppressed compared to naïve dimensional analysis in all other partial waves [3]: In the lth partial wave of the spin doublet ($\lambda = 1$) and quartet ($\lambda = -\frac{1}{2}$) channels, the amplitude converges for large off-shell momenta p as p^{-s_0-1}, $\mathrm{Re}[s_0] \geq 0$, with s_0 the solution to the algebraic equation

$$1 \;=\; (-)^l \, \frac{2^{1-l}\,\lambda}{\sqrt{3\pi}} \, \frac{\Gamma[\frac{l+1+s_0}{2}]\,\Gamma[\frac{l+1-s_0}{2}]}{\Gamma[l+\frac{3}{2}]} \times {}_2F_1\left[\frac{l+1+s_0}{2}, \frac{l+1-s_0}{2}; l+\frac{3}{2}; \frac{1}{4}\right] \quad .$$

Except in the $^2\mathrm{S}_{\frac{1}{2}}$ channel, the asymptotics is always weaker than in the naïve analysis ($s_0 = 1$). Thus, effective range corrections in the two-body system containing n external momenta need 3BFs with m derivatives to absorb UV divergences only when $n \geq m + \mathrm{Re}[2s_0]$. For example, the first Wigner-symmetric 3BF in the $^4\mathrm{S}_{\frac{3}{2}}$ channel is demoted from $\mathrm{N^4LO}$ to $\mathrm{N^7LO}$, suggesting that two-body physics rules this channel to very high accuracy.

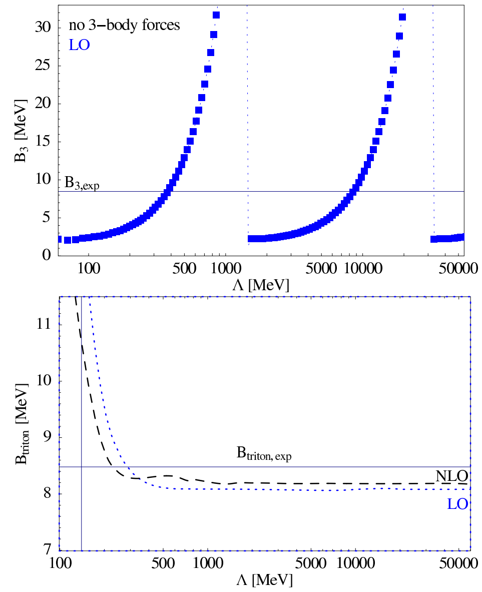

Fig. 1: Dependence of the triton binding energy on the cut-off Λ (top) without 3BF, (bottom) after the 3BF was tuned to the doublet-S scattering length. Notice the difference in scales.

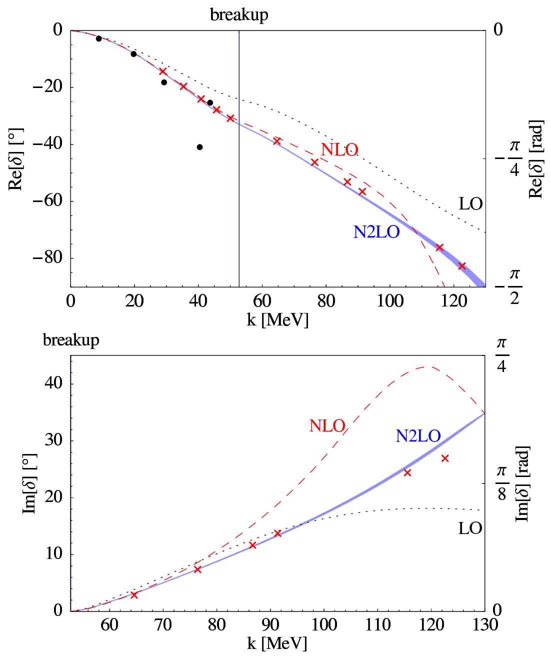

Fig. 2: The neutron-deuteron $^2\mathrm{S}_{\frac{1}{2}}$ phase shift at LO (dotted), NLO (dashed) and $\mathrm{N^2LO}$ (dark band from varying $\Lambda \in [200;\infty[$ MeV). Top: Real part. Bottom: Imaginary part. Dots: direct phase shift analysis from 1967; crosses: Argonne V18 potential + Urbana IX three-body forces.

References

[1] P. F. Bedaque, G. Rupak, H. W. Grießhammer and H.-W. Hammer: Nucl. Phys. **A714** (2003) 589 [nucl-th/0207034].
[2] B. van den Brandt, H. Glättli, H. W. Grießhammer, [nucl-ex/0401029]. Accepted for Nucl. Inst. Meth. **A**
[3] H. W. Grießhammer: *Suppressed Three-Body Forces at Very Low Energies*; forthcoming.

◇ Supported in part by DFG Sachbeihilfe GR 1887/2-1, 3-1 and BMBF