Decay Rate of Dissipative Many Body Systems in a Variational Approach

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In [1] based on the Perturbed Static Path Approximation (PSPA) [2] a formula for the decay rate of many body systems with separable two body interaction has been derived. The PSPA is known to break down at the so called "crossover" temperature T_0 and is not applicable for $T < T_0$. The reason for this (nonphysical) feature is that at T_0 mean field solutions become unstable (with frequencies of local modes becoming imaginary) such that fluctuations of large scale develop.

To overcome this difficulty we apply a novel approach to evaluate the partition function by a variational method [3] in application to many body systems with separable two body interactions [4]. This requires to know the eigen-modes of the many body system in the sense of a local RPA, thus determined by the secular equation $1 + k\chi(\varpi) = 0$. As this has many solutions, the full variational scheme is hardly applicable in full glory. For this reason one needs to reduce the complexity of the problem by restricting oneself to the relevant collective mode. This local mode can be described in terms of transport coefficients for inertia, friction and stiffness of [1], which depend on temperature $T = 1/\beta$ and vary with the collective coordinate q. The key point of the procedure is to define a reference action using these transport coefficients plus an *adjustable frequency* $\Omega(\beta, q)$. This $\Omega(\beta, q)$ turns out to be real for all q for which reason the previously mentioned T_0 problem does not show up.

As the variational approach to the partition function behaves completely regular at the crossover temperature T_0 and even far below it, we can take it as a starting point to derive a formula for the decay rate of a dissipative system from the imaginary part of the free energy, see [5,1]. If the system can be characterized by a free energy $\mathcal{F}(\beta, q)$ with only one minimum at $q = q_a$ and one barrier at $q = q_b$ we arrive at

$$R(\beta) = \frac{1}{\hbar\beta_0}\exp(-\beta\Delta\mathcal{F}(\beta))\sqrt{\frac{\mathcal{C}_\mathcal{F}(\beta, q_a)}{\mathcal{C}_\mathcal{F}(\beta, q_b)}}\frac{\zeta(\beta, q_b)}{\zeta(\beta, q_a)} \qquad (1)$$

for high temperatures $\beta < \beta_0 = 1/T_0$ and

$$R(\beta) = \frac{1}{\hbar\beta}\exp(-\beta\Delta\mathcal{F}(\beta))\sqrt{\frac{\mathcal{C}_\mathcal{F}(\beta, q_a)}{\mathcal{C}_\mathcal{F}(\beta, q_b)}}\frac{\zeta(\beta, q_b)}{\zeta(\beta, q_a)} \qquad (2)$$

for low temperatures $\beta > \beta_0$. In both formulas the barrier height $\Delta\mathcal{F}$ enters an Arrhenius factor and the stiffnesses $\mathcal{C}_\mathcal{F} = \partial^2\mathcal{F}/\partial q^2$ have to be taken at q_a and q_b. Note that (1) differs from the formula derived in [1] only in the definition of the correction factor that accounts for quantum fluctuations. There $\zeta(\beta, q_b) = \zeta^{PSPA}(\beta, q_b)$ showed a divergency at $\beta = \beta_0$ while here we use the regular $\zeta(\beta, q_b) = \zeta^{var}(\beta, q_b)$.

So far formulas like (1) and (2) with $\zeta(\beta, q) = \zeta^{var}(\beta, q)$ have not been presented in the literature, not even in the case of one dimension. Therefore, as a first step we have tested the new formula at the example of a particle in an asymmetric fourth order potential with barrier.

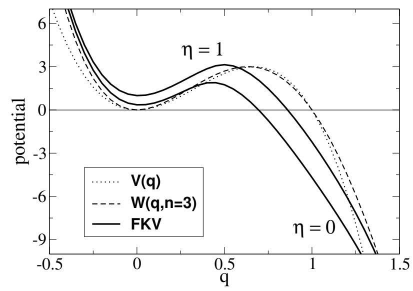

Fig. 1: Original potential $W(q)$ and the effective potential $W^{var}(q) = W^{FKV}(q)$ derived from the variational principle at $\beta = 2\beta_0$ for two different effective damping strengths η.

In Fig. 2 the logarithm of the decay rate of the system displayed in Fig. 1 is shown as function of $\beta = 1/T$. It is found [4] that at not too low temperatures the formulas (1) and (2) give very reasonable results. In the crossover region $\beta \approx \beta_0$ two versions of the variational approach (fully drawn) smoothly connect the PSPA rate to a more involved rate formula [6] which takes into account the full nonlinearity of the so-called "bounce-solution" (dashed). At very low temperatures the variational decay rate is unable to get the zero temperature limit right; it qualitatively behaves more like the classical rate and hence discards some quantum effects. The reason is that also the variational approach is based on a *static* approximation and therefore is not able to account for the *genuine time dependence* of the "bounce solution".

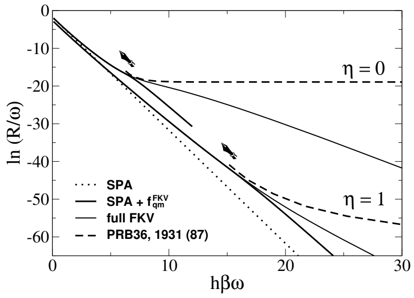

Fig. 2: Logarithm of the decay rate in various approaches for two different effective damping strengths η. The crossover β_0 is marked by an arrow.

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