A Nucleon in a Tiny Box ◇

H.W. Grießhammer

Chiral extrapolations of lattice "data" help to determine the coefficients of Chiral Effective Field Theory (χEFT) directly from QCD and reduce considerably the numerical effort in the computation of low-energy observables. Simulations are performed in boxes of Euclidean space-time, with finite-size effects controlled by $m_\pi L$, $m_\pi\beta$. L (β) is the spatial (temporal) lattice size and m_π the pion mass. Due to scale separation, many of the configurations in large lattices $L, \beta \gg 1/m_\pi$ describe pions travelling at large distances of the order of L, β. Therefore, a major part of computing-time is spent to simulate the well-understood Physics of the pion-cloud. It is however for $m_\pi L, m_\pi\beta \ll 1$ that this approach ist most profitable: While the pion-cloud feels the box, local interactions between nucleon-core and pions are unchanged for lattices bigger than the inverse breakdown-scale $\Lambda_\chi \approx 1$ GeV of χEFT. In the first extension of this "ϵ-régime" [2], $\frac{2\pi}{\Lambda_\chi} \lesssim L, \beta \lesssim \frac{2\pi}{m_\pi}$, to nucleons, we therefore computed the finite-volume effects on the nucleon mass to next-to-leading order (NLO) [1].

In the infinite volume, the pion is the Goldstone-boson of the spontaneously broken Chiral symmetry of QCD and describes *local* fluctuations around the preferred vacuum-direction, given by the chiral condensate $\langle\bar{q}q\rangle$ as order-parameter. The pionic zero-mode is frozen in at a particular vacuum expectation-value because the potential barrier to shift it around is proportional to $L^3\beta$, so that the tunnelling probability to other vacuum configurations is suppressed by $\exp -L^3\beta$. The condensate and the small quark-masses m_q are responsible for the non-zero pion-mass in the infinite-volume limit. In contradistinction, there is no spontaneous symmetry-breaking in a finite volume, and hence no chiral condensate, as the tunnelling probability increases exponentially with decreasing volume [2]:

$$m_\pi^2 \approx -2\frac{m_q}{f_\pi^2}\langle\bar{q}q\rangle \to m_{\pi,\mathrm{eff}}^2 = m_\pi^2\frac{\langle\bar{q}q\rangle_{L,\beta}}{\langle\bar{q}q\rangle} = m_\pi^2\frac{I_2(x)}{I_1(x)}, \quad (1)$$

with $x = \beta L^3 m_\pi^2 f_\pi^2$, f_π the pion decay-constant and $I_n(x)$ the nth modified Bessel function. The power-counting of χEFT is thus modified in the ϵ-régime: Albeit usually $m_\pi \approx \Delta \sim p$, we use $m_q \sim \sqrt{m_\pi} \sim \epsilon \sim \sqrt{\Delta}$ to reflect the dis-appearance of m_π. $\Delta \approx 270$ MeV is the $\Delta(1232)$-nucleon mass-splitting.

The pion-cloud around the nucleon changes thus non-trivially – besides being squeezed into a finite volume. While the contribution $-4m_{\pi,\mathrm{eff}}^2\ c_1$ of the quark-condensate to the nucleon-mass is the only LO effect in the infinite volume ($\sim p^2$), it disappears as ϵ^4 in the ϵ-régime. All other graphs in Fig. 1 contribute at the same order in the finite and infinite volume. As the pion-loops around the nucleon and Δ enter at ϵ^3 and p^3, respectively, they are promoted from NLO in the infinite volume to LO, and relativistic corrections (ϵ^4 and p^4) from N^{2}LO to NLO.

The coefficients c_2 and c_3 parameterise with c_1 the LO local iso-scalar nucleon-two-pion interactions:

$$\left[\frac{2m_{\pi,\mathrm{eff}}^2}{f_\pi^2}c_1\vec{\pi}^2 + \left(\frac{c_2}{f_\pi^2} - \frac{g_A^2}{8M_N f_\pi^2}\right)(\partial_0\vec{\pi})^2 + c_3\,(\partial_\mu\vec{\pi})^2\right] N^\dagger N \quad (2)$$

In principle, one can dis-entangle them in pion-nucleon scattering by different energy- and momentum-dependences. But their values bear large uncertainties as they are most sensitive to the small iso-scalar part of the amplitude. With explicit Δs, $c_1 = [-0.35 \pm 0.09]/\mathrm{GeV}, c_2 = [-1.49 \pm 0.67]/\mathrm{GeV}, c_3 = [0.93 \pm 0.87]/\mathrm{GeV}$ [3]. Their contribution is enhanced from N^{2}LO in the infinite-volume limit to NLO in the finite box. With the c_2-contribution negligible for $\beta/L \approx [1\dots 1.7]$, one can now fix the only free parameter, c_3, by lattice simulations of QCD. Figure 1 shows the sensitivity of the nucleon mass-shift on the box size when varying c_3 within the experimental accuracy. The result converges, and interpolates also to the infinite-volume limit. If the Δ is integrated out as heavy, the expansion converges poorly as c_2 and c_3 become large. The Δ must thus be retained explicitly. Relativistic corrections are negligible.

As the pion- (and hence quark-)mass dependence of the coupling strengthes is known in χEFT, simulations are greatly sped up by runs at heavier quark-masses, where effects from virtual quark-anti-quark loops are suppressed but the couplings do not change. Such a determination is simpler than pion-nucleon scattering on the lattice, as only three instead of five valence quarks are needed. It can be performed on lattices smaller than the inverse pion-mass. The χLF-collaboration (DESY/Zeuthen) will run simulations based on our findings [4].

References

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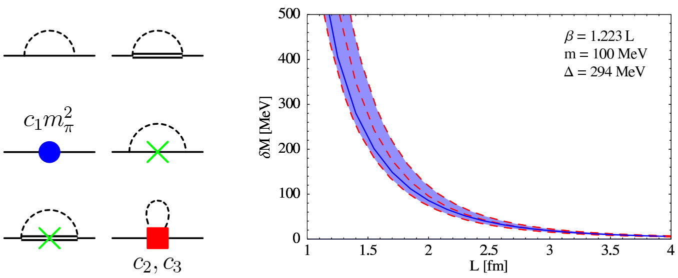

Fig. 1: Left: Contributions to the nucleon-mass change: Dashed, full, double lines: propagators of pionic non-zero modes, nucleon and Δ, respectively. Cross: relativistic correction; blob: coupling between nucleon and pionic zero-mode; square: pion-nucleon couplings of the NLO chiral Lagrangean. Right: Total mass-shift at LO (solid) and LO+NLO for $c_3 = \{0.06; 0.93; 1.8\}/\mathrm{GeV}$ (dashed, top to bottom; gray band: variation with c_3). From Ref. [1].

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