Chiral Extrapolations of Nucleon Properties ◊

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We analyze the quark mass dependence of the nucleon mass M_N and the axial-vector coupling constant g_A in the framework of the effective low-energy theory of QCD in the one-nucleon sector and two flavor case, namely $SU(2)$ Baryon Chiral Perturbation Theory (BChPT). The relevant formulae are compared with those obtained using an extension of this effective field theory which *explicitly* includes $\Delta\,(1232)$ degrees of freedom. A numerical analysis is performed taking as input lattice QCD data. It turns out that in order to obtain, at the one-loop level, an interpolating function able to connect the physical point with present lattice data at relatively large pion masses, the inclusion of explicit $\Delta\,(1232)$ degrees of freedom is not decisive for the nucleon mass but crucial for g_A.

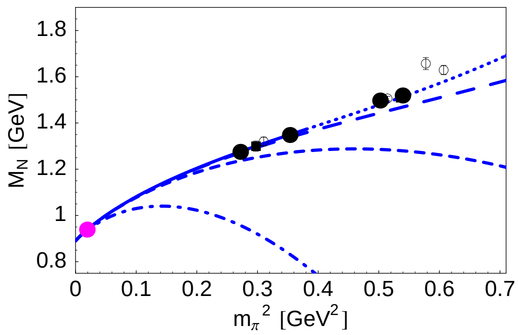

Fig. 1: Solid curve: best fit using the next-to-leading one-loop formula for $M_N(m_\pi)$ in BChPT without explicit $\Delta(1232)$ degrees of freedom. Dashed curve: leading one-loop result using for the unknown parameters the central values in the previous fit. The dark solid dots are CP-PACS data, the box refers to JLQCD and the empty circle to QCDSF-UKQCD simulations. The physical point is included as a constraint.

In [1] we worked out the quark mass dependence of M_N at the leading and next-to-leading one-loop level. We neglect isospin breaking effects and translate the functional dependence on the (light) quark mass m_q into a pion mass dependence according to the Gell-Mann-Oakes-Renner relation, consistently at the order we are working. We evaluate all the relevant graphs using the so-called infrared regularization method [2]. In the numerical analysis we estimated parameters not fixed by chiral symmetry by fitting the relevant formulae to a combined set of fully dynamical two-flavor lattice QCD data for $M_N(m_\pi)$ obtained by CP-PACS, JLQCD and QCDSF-UKQCD collaborations. In order to minimize artifacts from discretization and finite volume effects, we selected from the set of available points those with lattice spacing $a < 0.15\,\text{fm}$ and $m_\pi L > 5$, where L is the spatial size of the lattice. We assume that there is an overlap between the region in m_q where BChPT is applicable and the region of quark masses presently accessible to dynamical lattice QCD simulations, and explore the consequences of this assumption. We restricted ourselves to $m_\pi < 600\,\text{MeV}$. In the figure we plot the best fit (solid and dotted) curve coming from the next-to-leading one-loop expression. We include the physical point as a constraint.

As reported in [1] all the estimated parameters show high degree of consistency with information from low-energy πN scattering. The figure also shows the separate contributions coming from different powers in m_π when the result is expanded in the chiral limit.

For the pion-nucleon sigma term, applying the Feynman-Hellmann theorem to the pion mass dependence of M_N at the next-to-leading one-loop order, one finds $49 \pm 4\,\text{MeV}$ [1], in perfect agreement with the most often quoted $45 \pm 8\,\text{MeV}$.

Along the same line we analyzed $M_N(m_\pi)$ in the context of an effective field theory which apart from nucleons and pseudo-Goldstone bosons includes the $\Delta(1232)$ as an explicit degree of freedom [3]. This framework is an extension of $SU(2)$ BChPT with infrared regularization in the presence of spin-3/2 fields [4]. The delta-nucleon mass difference is treated as a small parameter and included in the power counting scheme. From the numerical study of the leading one-loop formula we can claim that an *explicit* treatment of the $\Delta(1232)$ is not essential for a satisfactory description of the pion mass dependence of the nucleon mass. We have also performed a careful error analysis associated to our fit procedure which confirms and sharpens all our previous conclusions [3].

Applying the same strategy to g_A, we have compared the chiral effective field theory schemes with and without explicit $\Delta\,(1232)$ degrees of freedom, both in their "nonrelativistic" [5] and manifestly Lorentz invariant form with infrared regularization [6]. We have evaluated the contributions up to leading and next-to-leading one-loop order in BChPT and used as input for the numerical analysis a set of (quenched) lattice data points provided by RIKEN-BNL-Columbia-KEK Collaboration. In the scheme without explicit $\Delta\,(1232)$ degrees of freedom, both at leading and next-to-leading one-loop level, we could not get a successful interpolating curve between lattice data and the physical point using parameters in agreement with low-energy πN scattering. However intermediate spin-3/2 resonance contributions are known to play a very important role in the axial properties of baryons. We have analyzed g_A at the leading one-loop order in the framework with explicit $\Delta\,(1232)$ degrees of freedom. In this framework, already at the leading one-loop level a satisfactory description of the pion mass dependence of g_A up to $m_\pi \approx 650\,\text{MeV}$ can be achieved.

References

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