Hot QCD at Finite Quark Chemical Potential: Lattice Thermodynamics and Field Theoretic Models ◇

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The study of QCD thermodynamics at finite chemical potential μ via lattice simulations is a formidable challenge, due to problems related to the non-reality of the euclidean Dirac operator when going to finite chemical potential. A lot of progress has recently been made, and the first lattice data at finite μ are now available. At the same time alternative strategies have been proposed, in order to improve our understanding of the QCD phase diagram. One of these is the study of two-colour QCD, a theory which has additional symmetries that guarantee the reality of the fermion determinant even at finite chemical potential.

We have investigated two- and three-colour QCD thermodynamics at finite μ by means of an extended Nambu Jona-Lasinio model including the scalar diquark channel. A background temporal gauge field couples the chiral and diquark condensates to the Polyakov loop via the gauge-covariant derivative. The Lagrangian of the model has the following form:

$$\mathcal{L}_{NJL}=\bar{\psi}\left(i\gamma_\mu D^\mu - m_0\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2+\left(\bar{\psi}\gamma_5\tau\psi\right)^2\right] + \frac{H}{2}\left(\bar{\psi}i\gamma_5\tau_2 t_2 C\bar{\psi}^T\right)\left(\psi^T C i\gamma_5\tau_2 t_2\psi\right) - V\left(\Phi\right), \quad (1)$$

where $V(\Phi)$ is the effective potential for the Polyakov loop field Φ. $V(\Phi)$ is a polynomial in Φ whose powers differ between the two- and three-color model, according to the different centre symmetries Z_2 and Z_3, and

$$D^\mu = \partial^\mu + igA^\mu, \qquad A^\mu = \delta^{\mu 0}A_0. \quad (2)$$

The Polyakov loop is related to A_0 by $\Phi = tr_c \exp\left[i\beta g A_0\right]$. The parameters of the Polyakov loop potential are fixed to lattice results in the pure gauge sector, while the NJL model parameters are fixed by reproducing the pion mass, pion decay constant and quark condensate at $T < T_c$.

From the Lagrangian (1), we can evaluate the thermodynamic potential of the system. Minimizing it, we can find the behaviour of the Polyakov loop and the chiral (σ) and diquark (Δ) condensates as functions of temperature and chemical potential. In the two-colour case, lattice data exist for the chiral and diquark condensates as functions of μ, at zero temperature. We show a comparison between our results [2] and the lattice data in the left panel of Fig. 1. We confirm that a second order phase transition occurs, from the vacuum to a state with finite baryonic density, at a critical chemical potential equal to half the mass of the Goldstone modes (pions and scalar diquarks). Another interesting quantity that can be evaluated in our model, and compared to the existing lattice data, is the pion mass as a function of the chemical potential, at zero temperature. Our results are shown in the right panel of Fig. 1 in comparison to the corresponding lattice data.

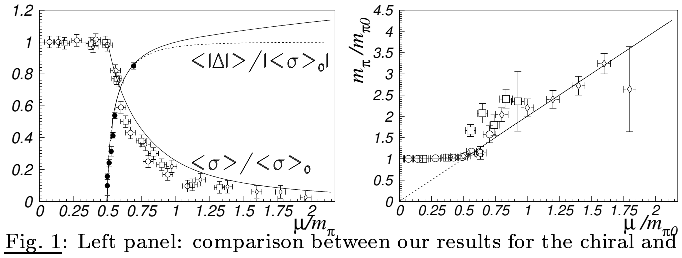

Fig. 1: Left panel: comparison between our results for the chiral and diquark condensates as functions of the chemical potential at zero temperature, and the corresponding lattice data taken from Ref. [1]; right panel: comparison between our results for the pion mass as a function of $\mu/m_{\pi 0}$ and the lattice data taken from Ref. [1].

In the three-colour case, lattice data at small chemical potential are available for two interesting quantities. The first one is the (scaled) quark number density which can be obtained from the thermodynamic potential of the system, $\Omega(T,\mu)$, through the relation

$$\frac{n_q}{T^3} = -\frac{1}{T^3}\frac{\partial\Omega(T,\mu)}{\partial\mu}. \quad (3)$$

The second one is the (scaled) pressure difference, defined as $\Delta p(T,\mu)/T^4 = (p(T,\mu) - p(T,\mu=0))/T^4$. We show our results [4] for these quantities in Fig. 2 in comparison with the corresponding lattice data.

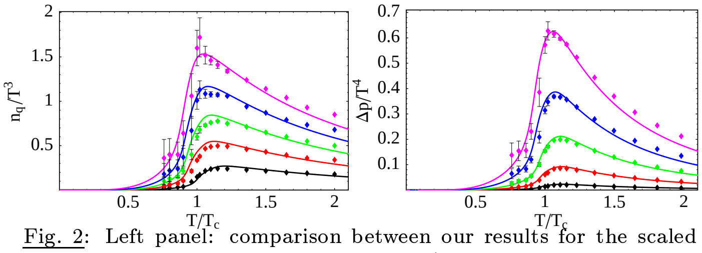

Fig. 2: Left panel: comparison between our results for the scaled quark number density as a function of T/T_c, and the corresponding lattice data taken from Ref. [3]. Right panel: comparison between our results for the scaled pressure difference as a function of T/T_c and the lattice data taken from Ref. [3]. In both panels, the different curves correspond to different quark chemical potentials: from bottom to top: $\mu = 0.2T_c$, $\mu = 0.4T_c$, $\mu = 0.6T_c$, $\mu = 0.8T_c$, $\mu = T_c$, with $T_c \simeq 170$ MeV.

From our results we can conclude that an NJL-type quasiparticle model, in which a coupling to a background temporal gauge field has been introduced, can reproduce successfully the existing lattice simulations of QCD thermodynamics at finite chemical potential, both for $N_c = 2$ and $N_c = 3$.

References

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