The Axial Coupling Constant of the Nucleon in Finite Volume ◇

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State of the art lattice QCD simulations of the hadron spectrum utilizing fully dynamical fermions can only be performed at rather large quark masses. Currently the mass of the lowest lying 0^- meson is typically larger than 500 MeV in such simulations. Comparing this value with the mass of physical pions $m_\pi = 140$ MeV, it is obvious that chiral extrapolations—i.e. the dependence of an observable like the axial coupling of the nucleon on the quark-masses employed in the simulation—are an important issue in lattice QCD. The methods of Chiral Effective Field Theory (ChEFT) can be utilized to address this question.

The axial coupling constant g_A of the nucleon has been studied theoretically as well as experimentally for many years. Its physical value $g_A = 1.2695$ is known to high accuracy from neutron beta decay experiments. Theoretically it can be defined as the value of the axial form factor of the nucleon at vanishing 4-momentum transfer. More explicitly, one considers the isovector axial current $A_\mu^{u-d} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$, where u and d denote the up and down quark fields. The proton matrix element of this current has the form factor decomposition

$$\langle p'|A_\mu^{u-d}|p\rangle = \bar{u}\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5\frac{q_\mu}{2m_N}G_P(q^2)\right]u\,. \quad (1)$$

Here $q = p' - p$ denotes the 4-momentum transfer and $u(p,s)$ is the proton spinor for momentum p and spin vector s. The axial coupling of the nucleon is then obtained via $g_A = G_A(q^2 = 0)$.

In ref. [1] it was shown that the quark-mass dependence of g_A calculated to leading one-loop order ($\mathcal{O}(\epsilon^3)$) in the SSE scheme [2] of Chiral Effective Field Theory provides an excellent description for the chiral extrapolation. In SSE one utilizes pions, nucleons and $\Delta(1232)$ states as the active degrees of freedom and organises the calculation perturbatively in powers of the parameter $\epsilon = \{|\vec{q}|, m_\pi, \Delta_0\}$, which can denote a small momentum $|\vec{q}|$, the pion mass m_π or the $N\Delta$ mass-splitting Δ_0. Given that the quark-mass dependence of the pion mass (squared) scales linearly with the quark-mass over a wide range of mass-parameters as observed in lattice QCD simulations, the problem of the quark mass dependence of g_A can be simplified to the question of the pion mass dependence of this quantity. In "free space" one obtains [1]

$$\begin{aligned} g_A(\infty) &= g_A^0 - \frac{(g_A^0)^3 m_\pi^2}{16\pi^2 F_\pi^2} + 4\{C(\lambda) + \frac{c_A^2}{4\pi^2 F_\pi^2}[\frac{155}{972}g_1 - \\ &\quad \frac{17}{36}g_A^0] + \gamma\ln\frac{m_\pi}{\lambda}\}m_\pi^2 + \frac{4c_A^2 g_A^0 m_\pi^3}{27\pi F_\pi^2\Delta_0} + \frac{8c_A^2 g_A^0}{27\pi^2 F_\pi^2} \\ &\quad m_\pi^2\sqrt{1-\frac{m_\pi^2}{\Delta_0^2}}\ln R + \frac{c_A^2\Delta_0^2}{81\pi^2F_\pi^2}\left(25g_1 - 57g_A^0\right) \\ &\quad \{\ln[\frac{2\Delta_0}{m_\pi}] - \sqrt{1-\frac{m_\pi^2}{\Delta_0^2}}\ln R\} + \mathcal{O}(\epsilon^4) \qquad (2) \end{aligned}$$

with $R = \Delta_0/m_\pi + \sqrt{\Delta_0^2/m_\pi^2 - 1}$. While most of the parameters in Eq.(2) are known from low energy physics [1], $g_A^0, C(\lambda)$ and g_1 are fit to the lattice data, where λ denotes the scale of dimensional regularization.

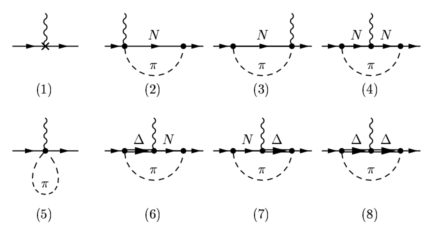

Fig. 1: Diagrams contributing to the quark mass dependence of the axial coupling constant to $\mathcal{O}(\epsilon^3)$ in the SSE scheme.

In ref. [3] the $\mathcal{O}(\epsilon^3)$ contributions to the axial coupling constant (Fig. 1) have now been evaluated in a finite box of length L with periodic boundary conditions. One obtains

$$g_A(L) = g_A(\infty) + \Delta g_A(L) + O(\epsilon^4)\,. \quad (3)$$

We note that the finite-volume-shift $\Delta g_A(L)$ depends only on parameters already known from the infinite volume result of Eq.(2) and can be expressed as an integral over Bessel-functions [3].

The resulting L-, respectively volume-dependence of $g_A(L)$ is shown in the lower part of Fig. 2 for a fixed pion mass of 600 MeV. The $\mathcal{O}(\epsilon^3)$ curve provides an excellent description for the volume-dependence when compared to actual lattice QCD data, further examples for different pion masses are given in [3].

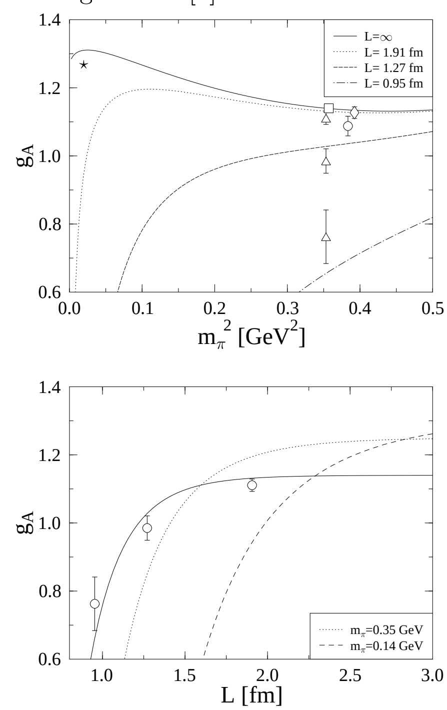

Fig. 2: Upper plot: Fit of the $\mathcal{O}(\epsilon^3)$ SSE extrapolation formula of Eq.(3) to the lattice data with pion masses below 650 MeV. Lower plot: Volume dependence of the lattice data for a fixed pion mass $m_\pi = 600$ MeV compared to the $\mathcal{O}(\epsilon^3)$ SSE prediction (solid curve) [3].

References

[1] T.R. Hemmert *et al.* Phys.Rev. **D68** (2003) 075009
[2] T.R. Hemmert etal, J.Phys. **G24** (1998) 1831
[3] A. Ali-Khan *et al.*, (QCDSF-UKQCD collaboration), preprint no. [hep-lat/0603028]; submitted to Phys. Rev. **D**.

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