Nucleon Mass: from Lattice QCD to the Chiral Limit ◇

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Lattice QCD on one side and chiral effective field theories on the other are progressively developing as important tools to deal with the non-perturbative nature of low-energy QCD and the structure of hadrons. At present, however, there is a gap between the relatively large quark masses accessible to full-QCD lattice simulations and the small quark masses relevant for comparison with physical nucleon properties. Systematic extrapolations guided by well-defined rules of low-energy QCD are necessary to bridge this gap. We study a way to merge Chiral Perturbation Theory, which predicts the quark mass dependence of nucleon observables, and lattice simulations, where the quark mass is a tunable parameter.

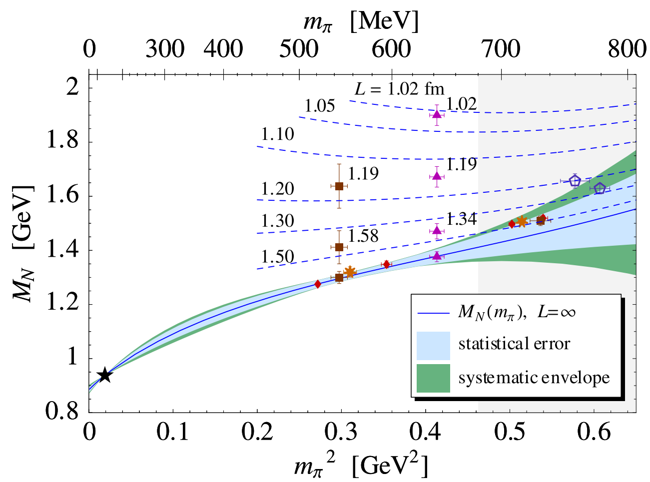
In ref. [1] the quark mass dependence (and consequently the pion mass dependence) of the nucleon mass M_N has been analyzed in the framework of Baryon Chiral Perturbation Theory (BχPT) with pions and nucleons as the active degrees of freedom. The calculation has been performed up to chiral order p^4 and infrared regularization [2] has been employed in the one-loop integrals. A remarkably good interpolation function between the most recent lattice data (see ref. [3,4]) and the physical nucleon mass has been obtained, with parameters in agreement with low-energy hadron phenomenology [1].

Ref. [5] updates and extends this investigation in several respects. The overall consistency of parameters and low-energy constants in the expansion of $M_N(m_\pi)$ with those extracted from pion-nucleon (πN) scattering is carefully examined. We demonstrate that the low-energy parameters of the chiral pion-nucleon effective Lagrangian which control the extrapolation are consistent with those extracted from πN and NN observables. In this context, $\Delta(1232)$ dominance in the relevant P-wave πN parameter has been clarified [5].

We have also studied the role of *explicit* $\Delta(1232)$ degrees of freedom in the one-loop self-energy. Unlike the situation with spin-dependent observables such as the nucleon axial coupling [6], we show that it is not crucial to introduce the $\Delta(1232)$ as a propagating degree of freedom when dealing with the mass of the nucleon. The off-shell propagation length of the Δ in the $\pi\Delta$ loop contribution to the nucleon self-energy is sufficiently short that its effects can be absorbed in low-energy constants, whose numerical values are indeed Δ-dominated [5].

We have also performed a detailed error analysis for the extrapolation from the lattice data through the physical point down to the chiral limit. Our study clearly shows the statistical significance and the stability of those extrapolations for pion masses below 0.6 GeV.

A highly useful additional source of information from lattice QCD results, apart from their quark mass (or m_π) dependence, is their variation with the finite spatial extent of the simulation volume L^3, see ref. [3]. Since the same BχPT framework describing how nucleon observables vary with the quark mass is also able to systematically account for finite size effects, we have combined the L- and m_π-dependence in our analysis of the nucleon mass in order to enlarge our input lattice data base [5], with obvious statistical benefits. Remarkably, also the results from the combined fits in m_π and L are in nice agreement with low-energy hadron phenomenology, confirming and sharpening the analysis in ref. [1].



Such a combined fit is presented in the figure above, where physical pion and nucleon mass (star) and data points from lattice QCD with pion masses smaller than 650 MeV serve as input. Chiral Perturbation Theory describes both the nucleon mass in infinite volume (solid curve, $L = \infty$) as well as the increase in mass due to a smaller simulation volume (dashed curves). Statistical errors in the lattice data lead to an uncertainty about the interpolation curve, shown here as the inner error band for the infinite volume nucleon mass. The surrounding darker band takes into account additional systematic uncertainties from fixed input parameters. The fit sharpens our knowledge about the nucleon mass in the chiral limit and about matrix elements such as the pion-nucleon sigma term.

In the future the winning strategy to perform precise chiral extrapolations of lattice data will be to analyze the combined quark mass and lattice spatial size dependence, *simultaneously* for several observables characterized by a common subset of low-energy constants.

References

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