

Chiral Effective Field Theory and Renormalization Group Methods \diamond

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Chiral effective field theories are important tools to describe the non-perturbative physics of QCD in the low-energy limit. In effective field theories, the dependence of the low-energy physics on high-energy degrees of freedom is encoded in low-energy coupling constants. Once these are determined, either from experiment or from non-perturbative calculations in the high-energy theory, the expansion in small external momenta of the low-energy degrees of freedom allows quantitative predictions of physical observables to a previously determined degree of accuracy.

In contrast, model systems for the low-energy physics in general do not aspire to provide accurate quantitative predictions, but to improve our qualitative understanding of physical mechanisms. Nevertheless, where more rigorous methods are not yet available, they can still provide useful quantitative results. An important example is QCD at finite density and temperature, where lattice simulations remain difficult. Even without gauge degrees of freedom, model systems such as the Nambu–Jona-Lasinio (NJL) model are still useful to explore the phase diagram of QCD at finite density and temperature. We have used such models to study the effects of a finite volume on low-energy observables such as the pion mass and the pion decay constant and on the chiral phase transition [1,2]. When gauge degrees of freedom are included by means of an effective potential, such models have been shown to describe QCD thermodynamics well at finite density and temperature [3].

In order to assess its reliability, we have studied the compatibility of a model system for the breaking of chiral symmetry and for the emergence of the pion-dominated low-momentum physics with chiral perturbation theory (χ PT).

In ref. [4] we have investigated the low-energy behavior of a bosonized version of the NJL-model with non-perturbative Renormalization Group (RG) flow equations and compare the quark mass dependence of the pion mass and the pion decay constant in this model to that in chiral perturbation theory. From this comparison we obtain effective values for two of the low-energy constants (Λ_3, Λ_4).

The main input into the numerical solution of the RG flow is the scale Λ_{UV} , where the RG evolution is started. This is similar to the usual cutoff scale for the NJL-model. The top figure for the pion decay constant shows the ratio f_π/F , the bottom figure the pion mass ratio $m_\pi^2/(M^2)$ as a function of the dimensionless current quark mass m_c/m_0 . $M^2 = m_c B$, F and B are lowest-order χ PT couplings, and $m_c/m_0 = 1$ is the physical point. The lines represent our results for values $\Lambda_{UV} = 1.0 - 1.5$ GeV in steps of 0.1 GeV, the shaded regions the one-loop χ PT-results from [5]. The results are compatible for a cutoff slightly above 1.0 GeV.

We have found that it is essential to include the renormalization of the pion propagator (as a wave function renormalization) in the RG scheme in order to obtain the quark mass dependence of the pion decay constant pre-

dicted by one-loop χ PT. We show that our RG scheme can be mapped onto a perturbative one-loop equation which reproduces the logarithmic dependence on the pion mass.

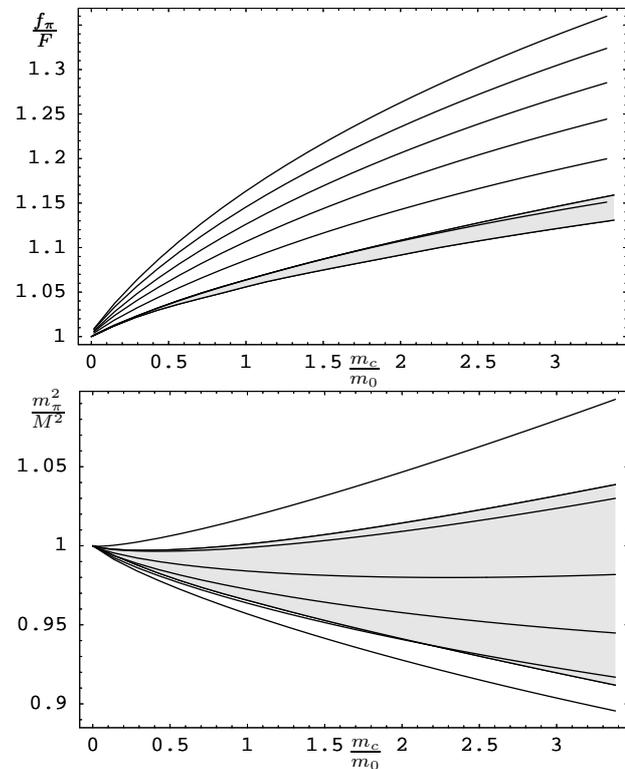


Fig. 1: The ratios f_π/F and $m_\pi^2/(m_c B)$ as functions of the scaled current quark mass m_c/m_0 . χ PT results are represented by the shaded regions, the RG results by the lines.

Thus, the comparison shows that for a particular range of parameters the model is compatible with those aspects of the low-energy physics that we have investigated, includes the relevant pion contributions and can be legitimately used here. However, it cannot be a model for many other aspects of low-energy QCD, as the existence of free, unconfined constituent quarks as low-energy degrees of freedom clearly demonstrates.

Beyond such model systems, non-perturbative RG methods are also applicable to gauge theories. In particular with regard to the phase transition in QCD, this approach has already yielded results and appears very promising.

References

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