In-medium Chiral Condensate beyond Linear Density Approximation ◇

N. Kaiser, P. de Homont, and W. Weise

The quark condensate $|\langle 0|\bar{q}q|0\rangle|$ is an order parameter of spontaneous chiral symmetry breaking in QCD. With increasing temperature and baryon density it decreases. For low T this effect can be systematically calculated in ChPT and the estimate $T_c \simeq 190\,\mathrm{MeV}$ for the critical temperature has been found. This is remarkably consistent with $T_c = (192 \pm 8)\,\mathrm{MeV}$ obtained in QCD lattice simulations (modulo still persisting disputes between different lattice groups).

The density dependence of the quark condensate can be extracted by exploiting the Feynman-Hellmann theorem with respect to the quark mass term $m_q\,\bar{q}q$. The leading linear term in the density ρ introduces the nucleon sigma-term $\sigma_N = \langle N|m_q\,\bar{q}q|N\rangle = m_q\,\partial M_N/\partial m_q = (45\pm 8)\,\mathrm{MeV}$. Corrections beyond it arise from the NN-interactions. Because of the Goldstone boson nature of the pion, $m_\pi^2 \sim m_q$, the pion-exchange dynamics in nuclear matter plays a particularly important role. Altogether, one has for the ratio of the in-medium to the vacuum chiral quark condensate:

$$\frac{\langle\bar{q}q\rangle(\rho)}{\langle 0|\bar{q}q|0\rangle} = 1 - \frac{\rho}{f_\pi^2}\left\{\frac{\sigma_N}{m_\pi^2}\left(1 - \frac{3k_f^2}{10M_N^2}\right) + D(k_f)\right\}, \qquad (1)$$

where interaction contributions are collected in the density dependent function

$$D(k_f) = \frac{1}{2m_\pi}\frac{\partial \bar{E}(k_f)}{\partial m_\pi}, \qquad (2)$$

defined as the derivative of the interaction energy per particle $\bar{E}(k_f)$ with respect to m_π^2. Our calculation [1] treats systematically the effects from one-pion exchange (with m_π-dependent vertex corrections), iterated 1π-exchange, and irreducible 2π-exchange with no, single and double $\Delta(1232)$-isobar excitations including Pauli-blocking corrections up to three-loop order.

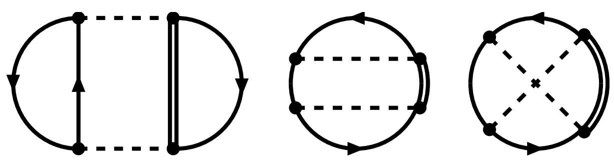

Figure 1: Three-loop Hartree and Fock diagrams of 2π-exchange with virtual $\Delta(1232)$-isobar excitation.

It is furthermore necessary to estimate the quark mass dependence of an NN-contact term which encodes short-distance dynamics. Employing recent computations of the NN-potential in lattice QCD [2] at different pion masses we find that the contact term has a negligible influence on the in-medium chiral condensate.

Fig. 2 shows the condensate ratio in the density region $0 \le \rho \le 0.36\,\mathrm{fm}^{-3}$ for three different values of the pion mass, $m_\pi = (0, 70, 135)\,\mathrm{MeV}$. One observes a very strong and non-linear dependence of the "dropping" condensate on the actual value of the pion mass m_π. In the chiral limit, $m_\pi = 0$, chiral symmetry seems to be restored already at about $1.5\rho_0$. This much faster decrease is caused primarily by the fact that the ratio:

$$\frac{\sigma_N}{m_\pi^2} = -4c_1 - \frac{9g_A^2 m_\pi}{64\pi f_\pi^2} + \frac{3c_1 m_\pi^2}{2\pi^2 f_\pi^2}\ln\frac{m_\pi}{\lambda} + \frac{9g_A^2}{(4\pi f_\pi)^2} \times\left\{\Delta\ln\frac{m_\pi}{2\Delta} + \sqrt{\Delta^2 - m_\pi^2}\,\ln\frac{\Delta + \sqrt{\Delta^2 - m_\pi^2}}{m_\pi}\right\}, \qquad (3)$$

is about 1.5 times larger in the chiral limit than at the physical point. By contrast, for the physical pion mass $m_\pi = 135\,\mathrm{MeV}$, the in-medium condensate stabilizes at about 60% of its vacuum value above that same density. In comparison to other works [3,4] we find more pronounced deviations from the linear density approximation above ρ_0.

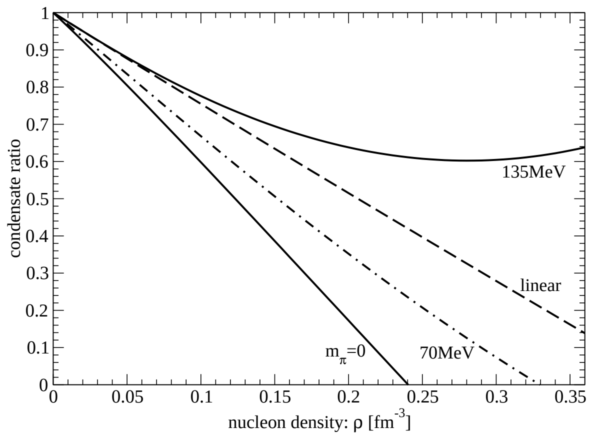

Figure 2: Ratio of in-medium to vacuum chiral condensate versus density ρ for three different values of the pion mass.

The non-linear density dependence of the chiral condensate is driven by 1π- and 2π-exchange processes at long and intermediate range for which the adequate framework is in-medium ChPT. In particular, the explicit treatment of 2π-exchange with virtual $\Delta(1232)$-excitation is mandatory for controlling the pion mass dependence of the energy per nucleon. The strong variation of the condensate with changing m_π, as seen in Fig. 1, points to the delicate interplay of spontaneous and explicit chiral symmetry breaking in fine-tuning the nuclear many-body problem. If the pion were a strictly massless Goldstone boson, chiral symmetry restoration would occur at a density so low that a description of nuclei in terms of nucleons and mesons would not be justified. The non-zero quark mass m_q of about 5 MeV turns out to be essential in order to stabilize the system.

References

[1] N. Kaiser, P. de Homont and W. Weise, Phys. Rev. **C77** (2008) 025204 and references therein.
[2] N. Ishii *et al.*, Phys. Rev. Lett. **99** (2007) 022001
[3] M. Lutz *et al.*, Phys. Lett. **B474** (2000) 7
[4] O. Plohl and C. Fuchs, Nucl. Phys. **A798** (2008) 75

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