Spin-orbit Interactions in Nuclei and Hypernuclei ◇

N. Kaiser and W. Weise

A detailed comparison is made between the spin-orbit interactions in nuclei and Λ hypernuclei. We argue that there are three contributions to the spin-orbit interaction:

- a short-range component involving scalar and vector mean fields
- a "wrong-sign" spin-orbit term generated by the one-pion exchange tensor force in second order
- a three-body term induced by two-pion exchange with excitation of virtual $\Delta(1232)$-isobars.

For nucleons in nuclei the long-range pieces related to the pion-exchange dynamics tend to cancel, leaving room dominantly for spin-orbit mechanisms of short-range origin (parametrized e.g. in terms of relativistic scalar and vector mean fields terms). In contrast, the absence of an analogous 2π-exchange three-body contribution for Λ hyperons in hypernuclei leads to an almost complete cancellation between the short-range (relativistic mean-field) component and the "wrong-sign" spin-orbit interaction generated by second order π-exchange with an intermediate Σ hyperon. These different balancing mechanisms between short- and long-range components can explain simultaneously the very strong spin-orbit interaction in ordinary nuclei and the weak spin-orbit splittings in Λ hypernuclei.

In order to quantify the spin-orbit term we consider the energy density functional. The spin-orbit coupling term:

$$\mathcal{E}_{\rm so}[\rho, \vec{J}] = F_{\rm so}(\rho)\, \vec{\nabla}\rho \cdot \vec{J}\,. \tag{1}$$

is constructed from the gradient of the nuclear density $\rho(\vec{r})$ and the spin-orbit density $\vec{J}(\vec{r})$. The empirical value of the spin-orbit coupling strength (as determined in the non-relativistic Skyrme phenomenology) is $F_{so}(\rho) \simeq 90\,\text{MeV fm}^5$. Scalar and vector boson exchange between nucleons with typical values for the squared ratios of coupling constants to boson masses, $G_S \simeq G_V \simeq 11\,\text{fm}^2$, reproduce this empirical value of $F_{so}(\rho) \simeq 90\,\text{MeV fm}^5$.

As a matter of fact the one-pion exchange tensor force in second order has a spin- and momentum-dependence such that it generates a spin-orbit term. We reproduce the expression for the dominant two-body Hartree contribution:

$$F_{\rm so}(\rho) = \frac{g_A^4 m_\pi M_N}{64\pi f_\pi^4}\left\{\frac{1}{m_\pi^2 + 4k_f^2} - \frac{3}{8k_f^2}\ln\frac{m_\pi^2+4k_f^2}{m_\pi^2}\right\}, \tag{2}$$

with $g_A \simeq 1.3$ and k_f the Fermi momentum related to the nucleon density by $\rho = 2k_f^3/3\pi^2$. The complete spin-orbit coupling strength from iterated 1π-exchange, with Fock term and Pauli blocking corrections included, is shown in the left part of Fig. 1. We demonstrate that this "wrong-sign" spin-orbit term gets compensated by three-body effects related to 2π-exchange with virtual $\Delta(1232)$ isobar excitation. The dominant Hartree term reads:

$$F_{\rm so}(\rho) = \frac{g_A^4}{8\pi^2\Delta f_\pi^4}\left\{\frac{m_\pi^2 k_f + 2k_f^3}{m_\pi^2+4k_f^2} - \frac{m_\pi^2}{4k_f}\ln\frac{m_\pi^2+4k_f^2}{m_\pi^2}\right\}, \tag{3}$$

with $\Delta = 293\,\text{MeV}$ the delta-nucleon mass splitting. The resulting three-body spin-orbit coupling strength is shown in the right part of Fig. 1. One sees that in the region around saturation density $\rho_0 = 0.16\,\text{fm}^{-3}$ both pieces of long-range origin tend to cancel each other.

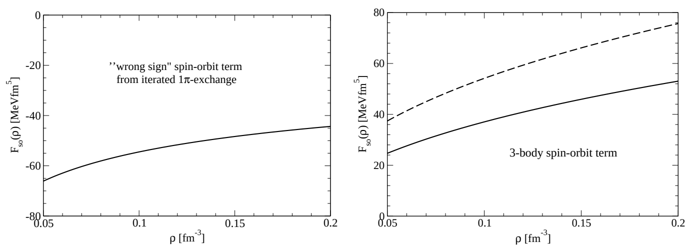

Figure 1: Nuclear spin-orbit coupling strength generated by the second order 1π-exchange tensor force (left) and the 2π-exchange three-nucleon interaction (right).

Next, we turn to the spin-orbit interaction of a Λ hyperon in hypernuclei. The pertinent quantity to extract the Λ-nuclear spin-orbit coupling is the spin-dependent part of the self-energy of a Λ hyperon interacting with weakly inhomogeneous nuclear matter:

$$\Sigma_{\rm spin} = \frac{i}{2}\vec{\sigma}\cdot(\vec{q}\times\vec{p})\,U_{\Lambda ls}(\rho)\,, \tag{4}$$

where $U_{\Lambda ls}(\rho)$ is taken in the limit of zero external Λ-momenta. The lower curve in Fig. 2 shows the "wrong-sign" Λ-nuclear spin-orbit coupling strength $U_{\Lambda ls}(\rho)$ generated by 2π-exchange with a Σ hyperon in the intermediate state. The upper two curves include in addition a short-range component which has been scaled to the one of nucleons, with $C_{ls} = 2/3$ and $1/2$ taken as options.

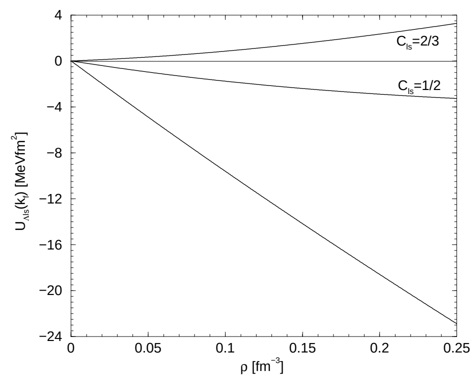

Figure 2: Spin-orbit coupling strength of a Λ hyperon.

References

[1] N. Kaiser and W. Weise, Nucl. Phys. A (2008), in print; nucl-th/0802.1190; and references therein.

◇ Work supported in part by BMBF, GSI, and the DFG Excellence Cluster "Origin and Structure of the Universe".