The Structure of the Nucleon in Lattice QCD ◇

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Transverse momentum dependent parton distribution functions (tmdPDFs) as well as generalized parton distributions (GPDs) play vital roles in our understanding of the structure of the nucleon in terms of the fundamental building blocks of QCD, the quarks and gluons. With the introduction of GPDs, it is possible not only to unify and extend the successful concepts of parton distribution functions and form factors, but also to give precise definitions of probability densities of quarks and gluons in impact parameter space and of such fundamental quantities as the quark and gluon angular momentum contributions to the nucleon spin [1]. Here, we present selected results on the nucleon spin structure [2] and the intrinsic transverse momentum of quarks [3], based on a lattice QCD study of nucleon matrix elements of local and non-local quark operators. The lattice simulations have been performed with $N_f = 2+1$ domain wall valence quarks and improved staggered sea quarks for pion masses as low as 350 MeV and volumes as large as $(3.5\ \mathrm{fm})^3$.

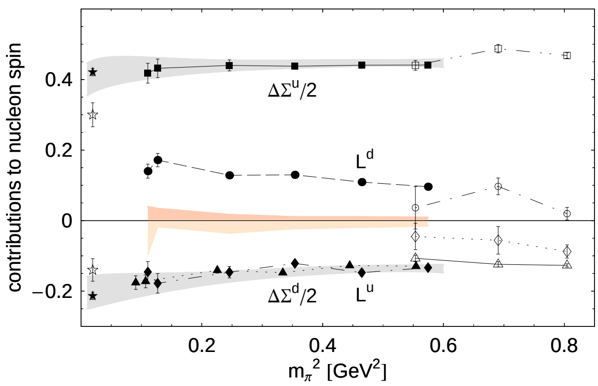

Figure 1: Quark contributions to the nucleon spin $S_z = 1/2$.

Starting from the QCD energy momentum tensor (EMT) and applying Noether's theorem, one obtains the nucleon spin sum rule $1/2 = (A(0) + B(0))/2$, where the EMT form factors $A(t)$ and $B(t)$ are directly related to the second x-moments of the GPDs $H(x,\xi,t)$ and $E(x,\xi,t)$, respectively. Introducing the quark spin fraction $\Delta\Sigma$, one can decompose the nucleon spin further, $1/2 = 1/2\Delta\Sigma + L^q + J^g$, where the orbital angular momentum (OAM) contribution of quarks to the nucleon spin is given by $L^q = (A^q(0)+B^q(0)-\Delta\Sigma)/2$. Our lattice calculation of the quark spin fraction $\Delta\Sigma$ and the EMT form factors A and B therefore allows us to extract the OAM contributions of quarks to the nucleon spin, which are shown in Fig. 1 as functions of the lattice pion mass. We find the amazing result that the individual up- and down-quark OAM contributions are large but cancel in sum, $L^{u+d} \approx 0$. Furthermore, the down quark spin and OAM contributions are of similar size but opposite in sign and therefore lead to a vanishing total contribution from down quarks to the nucleon spin, $J^d \approx 0$. Chiral extrapolations of the lattice data based on covariant BChPT [4] lead to similar conclusions at the physical pion mass. It is highly interesting to compare these results with first experimental determinations of $J^{u,d}$ from deeply virtual Compton scattering off the proton and the neutron at HERMES and JLab Hall A, respectively [5].

Similarly important insight into the nucleon structure is given by the tmdPDFs, which depend, in addition to the momentum fraction x, on the intrinsic transverse momentum $k_\perp$ carried by, e.g., the quarks in the nucleon. They not only play a central role in the description of semi-inclusive scattering experiments and related asymmetries, but, similarly to GPDs, have in general a probability density interpretation. For our Lattice QCD study of tmdPDFs, we employ non-local operators, $\mathcal{O}(l)$, build up from quark creation and annihilation operators which are spatially separated by a distance l and connected by a straight Wilson line. Nucleon matrix elements of such operators, $\langle P|\mathcal{O}(l)|P\rangle$, can be parametrized by a set of invariant complex amplitudes $A_i(l^2, l\cdot P)$. It turns out that the real part of the amplitude $A_2(l^2, l\cdot P)$ for $l\cdot P = 0$ is directly related to the lowest x-moment of the unpolarized (vector) tmdPDF, $f^{(n=1)}(k_\perp)$, by a Fourier transformation, $f^{(n=1)}(k_\perp) \propto \int d^2 l_\perp \exp(i l_\perp \cdot k_\perp)\mathrm{Re}A_2(-l_\perp^2, l\cdot P = 0)$.

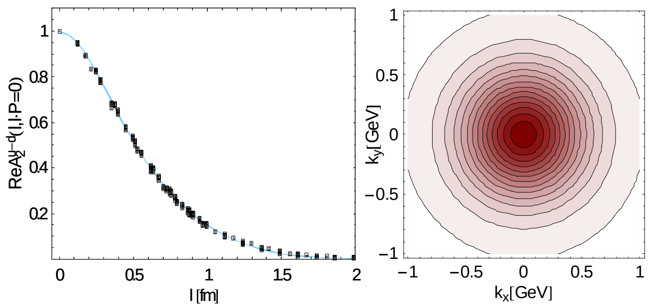

Figure 2: Real part of the amplitude A_2 and the resulting transverse momentum density in the isovector channel.

First results for the real part of A_2 and the corresponding probability distribution of quarks in the transverse momentum plane are shown in Fig. 2 for a lattice pion mass of ≈ 600 MeV. The amplitude, which has been normalized so that $\int d^2 k_\perp f^{(n=1)}_{u-d}(k_\perp) = 1$, shows a Gaussian-like distribution. From a single Gaussian fit, we find a root mean squared transverse momentum of $u-d$ quarks in the nucleon of $\langle k_\perp^2\rangle^{1/2} \sim 580$MeV, which is close to values used in the phenomenological description of, e.g., semi-inclusive deep inelastic scattering experiments.

References

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