Trace Anomaly of Nonlinear Electrodynamics ◇

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Nonlinear electromagnetism has several motivations, arising naturally from quantum electrodynamics (QED) and introduced by Euler and Heisenberg (EH) [1] in the first "effective" action. However, a more basic nonlinearity may be present since Maxwell's theory could be just the weak-field limit of a more fundamental sub-QED theory, just as Born-Infeld electrodynamics (BI) [2] is receiving today renewed attention as the physical limit of higher dimensional brane theories. Necessary to any nonlinear theory electromagnetism is a scale which has dimension of an electrical field, often expressed in terms of the electric field strength $eE_0 \equiv \frac{(Mc^2)^2}{\hbar c}$. The energy momentum tensor thus acquires a nonzero trace because the theory cannot be conformally symmetric, conversely to the well-known direction of the implication [3].

To see this, write the Lagrangian in terms of the Lorentz scalar $\mathcal{S} = (B^2 - E^2)/2$ and pseudoscalar $\mathcal{P} = E \cdot B$, but extracting the scale parameter, $V_{\text{eff}} \equiv M^4 V_{\text{eff}}\big(\frac{\mathcal{S}}{M^4}, \frac{\mathcal{P}}{M^4}\big)$. The infinite mass limit must naturally reproduce Maxwell electromagnetism $V_{\text{eff}}^{(\text{Max})} = -\mathcal{S}$. Now the energy-momentum tensor can be divided into a trace-less and a trace part, the latter of which takes the form

$$T^\mu_\mu = 4\left(V_{\text{eff}} - \mathcal{S}\frac{\partial V_{\text{eff}}}{\partial \mathcal{S}} - \mathcal{P}\frac{\partial V_{\text{eff}}}{\partial \mathcal{P}}\right) = M\frac{\partial \overline{V}_{\text{eff}}}{\partial M}. \qquad (1)$$

The introduction of the bar signals the essentially nonlinear parts of V_{eff}: terms linear in $\mathcal{S}$ are seen to cancel explicitly in the preceding expression. This suggests the decomposition $V_{\text{eff}} = V_{\text{eff}}^{(1)} + \overline{V}_{\text{eff}}$, using $V_{\text{eff}}^{(1)}$ to denote the remaining terms linear in $\mathcal{S}$.

BI is a classical nonlinear theory constructed to have a limiting field strength, of which a nonvanishing T^μ_μ is an immediate consequence. An easy calculation shows

$$T^\mu_\mu = 4M^4(\varepsilon(1 + \mathcal{S}/M^4 1) - 1) \quad (\text{BI}), \qquad (2)$$

where $\varepsilon \equiv -(\partial V_{\text{eff}}^{(\text{BI})}/\partial \mathcal{S}) = [1 + 2\mathcal{S}/M^4 - (\mathcal{P}/M^4)^2]^{-1/2}$ is the polarization function. Eq. (2) is a smooth, analytic function on its usual domain of definition, $E, B < E_0$. The explicit nonanalyticity at the critical field parallels the divergence of thermodynamic potentials at a phase transition; however, in the absence of the more fundamental theory displaying both phases, we restrain ourselves to using BI as a comparison to non-perturbative investigations of gauge theories, for which a phase transition is universally expected.

In QED, the natural scale m_e of electron-positron fluctuations induces violations of superposition and other nonlinearities in light propagation even at one loop in perturbation theory. Recalling the well-known relation $m(dV_{\text{eff}}/dm) = -m\langle\bar\psi\psi\rangle$ as well as the decomposition suggested below Eq. (1), one finds (paralleling QCD)

$$T^\mu_\mu = \frac{2\alpha}{3\pi}\langle\mathcal{S}\rangle + m\langle\bar\psi\psi\rangle \quad (\text{QED}), \qquad (3)$$

writing the vacuum expectation of $\mathcal{S}$ due to the interpretation of the linear term of the effective action to be the externally applied field or photon condensate.

The derivative d/dm regularizes all but the zero-point divergence of the Euler-Heisenberg effective action, so a meromorphic expansion transforms the expressions for the e^+e^- condensate and trace anomaly into integrals more amenable to numerical evaluation. For example in electric-only background

$$T^\mu_\mu = \frac{m^4}{2\pi^2\beta}\int_0^\infty \frac{s^2\ln(1 - e^{-\beta s})}{1 - s^2 - i\epsilon}ds \qquad (4)$$

and

$$-m\langle\bar\psi\psi\rangle = \frac{m^4}{2\pi^2\beta}\int_0^\infty \frac{\ln(1 - e^{-\beta s})}{1 - s^2 - i\epsilon}ds \qquad (5)$$

in which the imaginary part generated by the pole persists as the image of the instability of the vacuum to pair production. These results are readily generalized to arbitrary, constant background fields and to scalar electrodynamics [3]. The EH anomaly exhibits a striking edge architecture (see figure), deserving further investigation.

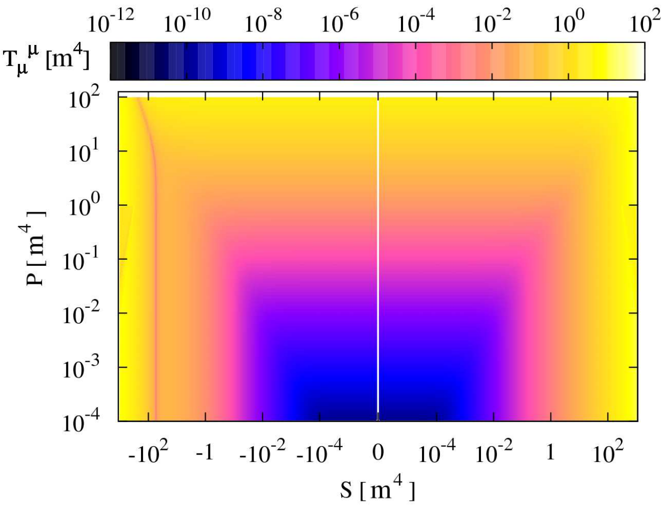

Fig. 1: The trace anomaly of the EH action for general E,B fields, parameterized by the Lorentz invariants, $\mathcal{S}, \mathcal{P}$. Although the figure has a logarithmic scale, the anomaly crosses zero at the dark (pink) line, going from positive to negative for $\mathcal{S} \lesssim -55$. (Color online.)

The trace anomaly enters Einstein's equations in the manner of a cosmological constant, showing that where present, it tends to push matter apart. Because a perfectly field-free space is difficult to imagine, large-scale gravitational consequences could be nontrivial despite the small magnitude, and the effects of 'clumpy' dark energy are only recently being investigated. Potential implications include modifications to orbital dynamics and astrophysical collapse dynamics.

References

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