Monte-Carlo Simulations of QCD Thermodynamics in the PNJL Model ◇

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Results of QCD thermodynamics from lattice computations are reproduced surprisingly well with a quasiparticle model, an extension of the Nambu – Jona-Lasinio model with inclusion of Polyakov loop dynamics (the PNJL model) at the mean field level [1,2,3]. A better understanding of these results requests the investigation of fluctuations beyond mean field. This can be done by numerical simulations of the thermodynamics using standard Monte-Carlo techniques (MC-PNJL). The advantage over other methods is that Monte-Carlo calculations automatically incorporate fluctuations to all orders.

In order to introduce fluctuations in the PNJL model we consider the total volume of our system as composed of a certain number of subregions, $V = \sum_i V_i$, and require that the fields are completely correlated if they lie in the same subvolume V_i and decorrelated otherwise [4]. In this way the fields can fluctuate in the sense that they can have different values from one subvolume to another. Physically this means that our fields are only weakly dependent on the position and can be considered constant inside subvolumes of a given dimension.

We fix $V_i = \frac{k}{T^3}$ (with temperature T) for every V_i and calculate different observables using standard Metropolis algorithm for different values of the parameter k. This parameter is varied in the range $64 \leqslant k \leqslant 2500$, the lower limit corresponding to the volume used in lattice simulations. I.e. there is a factor of ~ 40 between the smallest and the largest volume. In this way we can study the dependence of the observables as a function of the volume size.

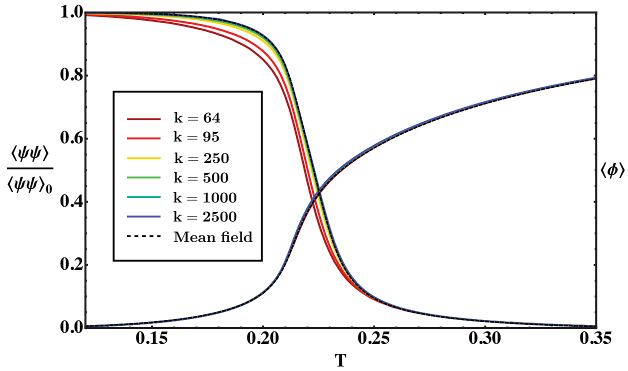

Fig. 1: Chiral condensate $\langle\bar\psi\psi\rangle$ and Polyakov loop $\langle\phi\rangle$: dependence on the volume size. Small deviations from the mean-field result are manifest only for the chiral condensate. The behavior of the Polyakov loop is completely unchanged.

Here we focus on the case $N_f = 2$. We first study how the chiral and deconfinement transitions are affected by the introduction of fluctuations. This can be achieved evaluating the chiral condensate and the Polyakov loop for different volumes and comparing this with the mean-field result. This comparison is presented in Fig. 1: the presence of fluctuations does not modify the behavior of the Polyakov loop. All sets of data for different k overlap perfectly. For the chiral condensate we notice only weak dependence on k below the critical temperature. We conclude that the order parameters of the transitions are not affected by the presence of fluctuations.

Unlike the chiral condensate and the Polyakov loop, flavour non-diagonal susceptibilities such as the second derivative of the pressure p with respect to $u-$ and $d-$quark chemical potentials,

$$\chi_{ud} = 2T^2\, c_2^{ud} = \frac{\partial^2 p}{\partial\mu_u \partial\mu_d}\ , \tag{1}$$

taken at $\mu_u = \mu_d = 0$, are particularly sensitive to the presence of fluctuations. It is remarkable that, while the mean-field PNJL model predicts a vanishing c_2^{ud}, the Monte-Carlo results agree very well with the available lattice data as shown in Fig. 2. This effect is related to the presence of fluctuations with pion quantum numbers in the Monte-Carlo computations.

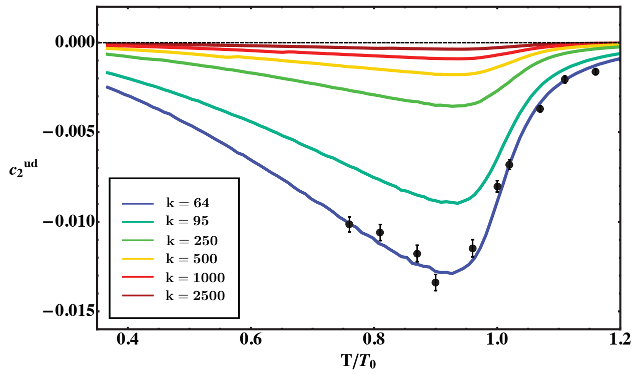

Fig. 2: Second order flavour non-diagonal coefficient c_2^{ud} in comparison with lattice data [5]

References

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