The Interplay of Flavour and Colour in PNJL Models ◇

S. Rößner, T. Hell, and W. Weise

The Polyakov loop extended Nambu and Jona-Lasinio model (PNJL model) in a mean field framework shows astonishingly good agreement with lattice QCD calculations [1,2]. This is illustrated in Fig. 1, where lattice and PNJL results for the flavour diagonal moments of the normalized pressure are compared. The flavour diagonal and off diagonal moments of the normalized pressure p/T^4 are defined in Eq. (1) and (2) in agreement with Ref. [4].

$$c_n^{\mathrm{uu}} = \frac{1}{n!}\,\frac{\partial^2}{\partial^2(\mu_\mathrm{u}/T)}\,\frac{\partial^{n-2}(p/T^4)}{\partial^{n-2}(\mu/T)}\bigg|_{\mu_u=\mu_d=\mu=0} \tag{1}$$

$$c_n^{\mathrm{ud}} = \frac{1}{n!}\,\frac{\partial}{\partial(\mu_\mathrm{u}/T)}\,\frac{\partial}{\partial(\mu_\mathrm{d}/T)}\,\frac{\partial^{n-2}(p/T^4)}{\partial^{n-2}(\mu/T)}\bigg|_{\mu_u=\mu_d=\mu=0} \tag{2}$$

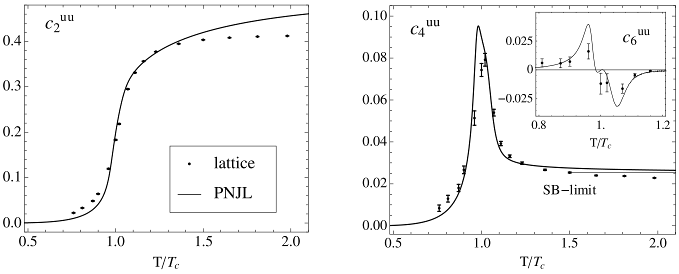

Fig. 1: A comparison of flavour diagonal moments of the normalized pressure evaluated on the lattice [4] and in the PNJL model.

The approach beyond mean field theory advocated in Ref. [3] is able to implement Polyakov-loop degrees of freedom correctly allowing for differences in $\langle\Phi^*\rangle$ and $\langle\Phi\rangle$ at $\mu \neq 0$ plotted in the inset of the left panel of Fig. 2. The comparison with c_2^{ud} defined in Ref. [4] and plotted on the left hand side of Fig. 2 suggests a common origin of c_2^{ud} and $\Phi^- = \frac{1}{2}\langle\Phi^* - \Phi\rangle$.

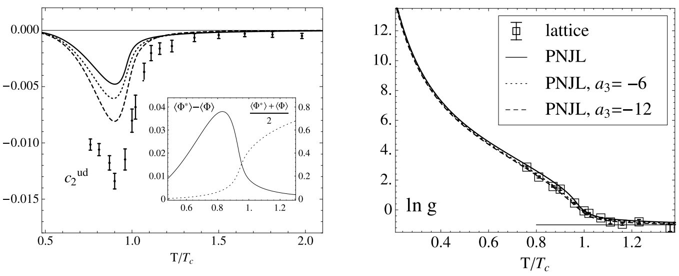

Fig. 2: Left: The isovector moment c_2^{ud} and, in the inset, the Polyakov-loop degree of freedom $\Phi^- = \frac{1}{2}\langle\Phi^* - \Phi\rangle$ at $\mu > 0$ show large similiarities. Softer Polyakov loop potentials in Φ^- increase $|c_2^{\mathrm{ud}}|$ (dotted and dashed lines).
Right: Effective isovector coupling g of flavoured quark densities to Φ^- extracted from PNJL and lattice QCD calculations. The effective isovector coupling g is stable under changes to the Polyakov loop effective potential.

The schematic partition function

$$\mathcal{Z}(\mu_\mathrm{u},\,\mu_\mathrm{d},\,\xi) = \mathcal{Z}_\mathrm{u}(\mu_\mathrm{u})\cdot\mathcal{Z}_\mathrm{d}(\mu_\mathrm{d})\cdot\mathcal{Z}_\mathrm{I}(\mu_\mathrm{u},\,\mu_\mathrm{d},\,\xi)\ , \tag{3}$$

models an interconnection of colour and flavour. In this describtion up and down quark contributions $\mathcal{Z}_\mathrm{u}(\mu_\mathrm{u})\mathcal{Z}_\mathrm{d}(\mu_\mathrm{d})$ factorize completely, leading to vanishing up-down quark susceptibilities χ_{ud}. The interaction part

$$\log\mathcal{Z}_\mathrm{I}(\mu_\mathrm{u},\,\mu_\mathrm{d},\,\xi) = VT^3\,g\,\xi\left(\frac{n_\mathrm{u}}{T^3}+\frac{n_\mathrm{d}}{T^3}\right) \tag{4}$$

which is treated perturbatively couples the flavoured quark densities to the field ξ. It is this perturbative part $\mathcal{Z}_\mathrm{I}$ that induces non-vanishing flavour off diagonal susceptibilities:

$$\frac{\chi_{\mathrm{ud}}}{T^2} = \frac{\chi_{\mathrm{uu}}^{(0)}}{T^2}\,\frac{-g\,(\mathrm{d}\xi/\mathrm{d}(\frac{\mu}{T}))}{1+2g\,(\mathrm{d}\xi/\mathrm{d}(\frac{\mu}{T}))}\ , \tag{5}$$

where $\chi_{\mathrm{uu}}^{(0)}$ is the flavour diagonal susceptibility of the unperturbed schematic model. Eq. (5) and Fig. 2 suggest to identify ξ with $\Phi^- = \frac{1}{2}\langle\Phi^* - \Phi\rangle$ in the PNJL model. Alternatively the flavour off diagonal susceptibilty may be expressed in terms of the effective potential Ω

$$\chi_{\mathrm{ud}} = -\frac{1}{2}\frac{\partial^2\Omega}{\partial\mu\partial\xi}\left[\frac{\partial^2\Omega}{\partial\xi^2}\right]^{-1}\frac{\partial^2\Omega}{\partial\mu\partial\xi}\ . \tag{6}$$

To consolidate the study of the PNJL model behaviour the effective Polyakov loop potential has been modified. The potential used in Refs. [1,3] may be supplemented by an additional higher order term term in the polynomial ansatz of the coefficient $a(T)$. The coefficient a_3 of the additional term $a_3\,(T_0/T)^3$ has been fixed to the values -6 and -12. The ratio of c_2^{ud} and $\mathrm{d}\Phi^-/\mathrm{d}\mu$ is almost flat around T_c and remains almost unaffected by changes to the potential. We take this as a confirmation of the identification of ξ with Φ^-. The changes to the effective Polyakov loop potential are changes to the stiffness of the potential in Φ^--direction around T_c. The effects of softer Polyakov loop potentials are illustrated by the dotted and dashed lines in Fig. 2. We argue that it is therefore possible to extract information about an effective Polyakov loop potential from flavour off diagonal quantities.

Such a scenario allows to extract the isovector coupling constant g from both PNJL and lattice QCD (see right panel of Fig. 2). The determination of the extend to which the presented mechanism is at work in QCD requires further investigation of the current quark mass and strangeness dependence.

References

[1] S. Rößner, C. Ratti and W. Weise, Phys. Rev. **D75** (2007) 034007 [arXiv:hep-ph/0609281].

[2] C. Ratti, S. Rößner and W. Weise, Phys.Lett. **B649** (2007) 57 [arXiv:hep-ph/0701091].

[3] S. Rößner, T. Hell, C. Ratti and W. Weise, Nucl. Phys. **A814** (2008) 118 [arXiv:0712.3152 [hep-ph]].

[4] C. R. Allton *et al.*, Phys.Rev. **D 71** (2005) 054508 [arXiv:hep-lat/0501030].

◇ Work supported in part by BMBF, GSI, the DFG excellence cluster “Origin and Structure of the Universe” and by the Elitenetzwerk Bayern.