Chiral Thermodynamics of Nuclear Matter \diamond

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In the framework [1] of the in-medium Chiral Perturbation Theory (ChPT) we study the nuclear matter equation of state at finite temperatures T and variable proton-toneutron ratio ρ_p/ρ_n .

In previous works [2,3] a consistent picture for chiral dynamics of nuclear matter has been established. In these calculations pions, nucleons and Δ -isobar excitations have been considered as explicit degrees of freedom. The corresponding diagrammatic contributions to the energy density have been evaluated up to three-loop order. At finite temperatures T the contributions to the free energy arise from two-body terms (1 π - and 2 π -exchange nucleon-nucleon interactions), three-body terms (Pauli-blocking effects and 2 π -exchange three-body forces), and contact terms which account for unresolved short-distance dynamics.

We have verified that a good representation of the resulting equation of state can be constructed by means of a third-order virial expansion of the pressure: $P(\rho, T) =$ $T \rho \left[1 + B(T) \rho + C(T) \rho^2\right]$. The second and third virial coefficients B(T) and C(T) reflect the two- and three-body correlations in the nuclear medium, respectively. They depend on temperature and approach zero at high temperatures (ideal gas limit). On the contrary at low temperatures the virial coefficients B(T) and C(T) are large in magnitude, since one is dealing with a strongly interacting system. It is remarkable that this simple parameterization of the equation of state can represent all the relevant many-body correlations induced by chiral pion exchange.

The resulting equation of state $P(\rho, T)$ in the case of isospin-symmetric nuclear matter is shown in Fig. 1. The curves display a first-order liquid-gas phase transition with a critical temperature of $T_c \simeq 15$ MeV. At zero temperature the minimum of the energy per particle is -16.0 MeV at $\rho_0 \simeq 0.16$ fm⁻³, reproducing the known saturation properties of nuclear matter [3].



Fig. 1: Pressure isotherms of isospin-symmetric nuclear matter at different temperatures. The first-order liquid-gas phase transition region extends over a wide range of densities and temperatures. The critical point is located at $\rho_c \simeq 0.053 \text{ fm}^{-3}$ and $T_c \simeq 15 \text{ MeV}$.

The present work extends the isospin-symmetric case to a variable proton-to-neutron ratio. In order to account for the isospin-asymmetry, it is sufficient to recalculate the isospin factors of the diagrams contributing to the free energy density as a function of the neutron fraction N/(N+Z).

An example for the results so obtained is displayed in Fig. 2. It shows the equation of state $P(\rho, T)$ for neutronrich matter with a neutron fraction of 0.9. The coexistence region of the liquid and gas phases disappears almost and the critical temperature decreases to about 6.5 MeV. The pressure grows now faster with density in comparison to the previous isospin-symmetric case, as expected from the Pauli exclusion principle and the smaller degeneracy factor. At T = 0 the energy per particle is always positive indicating no self-binding in the neutron-rich system.



Fig. 2: Pressure isotherms of isospin-asymmetric nuclear matter at different temperatures. The matter consists of 90% of neutrons and 10% of protons. The region of the first-order liquid-gas phase transition is strongly reduced: $T_c \simeq 6.5$ MeV.

Applications of our equation of state to astrophysical problems are currently under investigation. They include a detailed comparison with the equation of state of Akmal-Pandharipande-Ravenhall [4] based on a sophisticated many-body calculation. The range in density where the present perturbative calculation is valid is $\rho \lesssim 0.2 \,\mathrm{fm}^{-3}$, corresponding to the crust and the external region of the core of a neutron star.

References

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